Generating accurate jitter for SERDES-receiver-tolerance testing

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In some design situations, sophisticated equipment is unavailable, or an engineer wants to use a simple setup to quickly characterize a device. This article presents a simple method for generating accurate jitter numbers. The generated jitter is entirely sinusoidal and therefore deterministic, and you could use it to plot the jitter-tolerance curve of a serial high-speed receiver.

The article emphasizes the method of generating the jitter and the theory behind it as well as lab experiments meant to verify the theory and instill confidence in the method’s viability. The text uses clock and carrier frequency interchangeably.

Frequency-modulated signals and derivation of equivalent jitter

The noise we are interested in on the clocks we are dealing with is mostly FM (frequency-modulation)-related. What counts is the deviation of the clock transition point from the ideal transition time instant. Any deviation from the ideal transition point translates into a period error. The frequency of a signal relates closely to its period, so varying the period will result in a frequency change. Studying an FM signal should reveal a relationship between the FM characteristics and the amount of jitter that the FM induces.

The general math representation of a FM signal $x(t)$ is:

$$x(t) = A_0 \cdot \cos(\omega_0 t + a \cdot \int m(t) \cdot dt),$$

where $x(t)$ is the frequency-modulated signal, $m(t)$ is the modulating signal, $a$ is a proportionality constant, and $A_0$ is the amplitude of the FM signal.

You can calculate the instantaneous frequency of the signal by taking the first derivative of the phase with regard to the time; that is:

$$\omega(t) = \frac{d}{dt} (\varphi(t)) = \frac{d}{dt} (\omega_0 \cdot t + a \cdot \int m(t) \cdot dt) = \omega_0 + a \cdot m(t),$$

where $\omega(t)$ is the instantaneous frequency of the signal, and $\varphi(t)$ is the instantaneous phase of the signal.

To simplify the analysis, consider the harmonic case—that is, $m(t)$ is a sinusoidal function, $m(t) = \cos(\omega_m t)$. The phase of the FM signal becomes:

$$\varphi(t) = \omega_0 \cdot t + \Delta \omega \cdot \int \cos(\omega_m \cdot t) \cdot dt \ ; \ \Delta \omega$$ is the frequency deviation.

You can then write the frequency-modulated signal as:

$$x(t) = A_0 \cdot \cos(\omega_0 t + \frac{\Delta \omega}{\omega_m} \cdot \sin(\omega_m t)) = A_0 \cdot \cos(\omega_0 t + \beta \cdot \sin(\omega_m t)),$$

where $\beta = \Delta \omega / \omega_m$ is the modulation index known from the frequency-modulation theory. For simplicity, consider the initial phases of the main carrier and of the modulating signal equal to zero, which does not impact the end result.

If you know the central frequency $\omega_0$, you could translate the phase into actual time delay or deviation of the zero crossing point of the modulated wave from the zero
crossing of the nonmodulated carrier. The term of the phase that generates the phase deviation from the pure carrier is the integral of the modulating signal. To estimate the maximum deviation or, in other words, the equivalent peak-to-peak jitter the frequency modulation induces, you need to estimate the maximum value that the integral can take. Integrating the cosine between any \(-\pi/2+2k\pi, \pi/2+2k\pi\) interval will determine the maximum amount of positive phase deviation, because the cosine is positive inside that interval. The term that reflects the phase advance or delay is:

\[
\beta = \frac{\Delta \omega}{\omega_m} \cdot \int_{-\pi/2}^{\pi/2} \cos(\omega_m \cdot t) \, dt = \frac{\Delta \omega}{\omega_m} \cdot \sin(\omega_m \cdot t) \bigg|_{-\pi/2}^{\pi/2} = 2 \cdot \frac{\Delta \omega}{\omega_m} = 2 \cdot \beta.
\]

The contribution of the negative side of the modulating cosine \((\omega_m)\) will be identical in magnitude; therefore, the total peak-to-peak phase deviation will be 4\(\beta\) radians.

To translate the result from radians to actual time delay, use the fact that a full clock period is equivalent to 2\(\pi\) radians.

\[
delay = \frac{1}{2 \cdot \pi \cdot f_0} \left[ \frac{s}{\text{rad}} \right].
\]

To determine the peak-to-peak jitter that the sinusoidal frequency modulation induces, combine equations 5 and 6:

\[
T_{\text{delay}}[\text{seconds peak to peak}] = \frac{4 \cdot \beta}{2 \cdot \pi \cdot f_0} = \frac{2 \cdot \beta}{\pi \cdot f_0}.
\]

Equation 7 shows that the amount of time jitter that the frequency modulation induces simultaneously depends on the carrier frequency \((f_0)\), the frequency deviation \((\Delta \omega)\), and the modulating frequency \((\omega_m)\). Also, if the carrier frequency is held constant, for the jitter to be unchanged at different modulation rates \((\omega_m)\), the modulation index \((\beta)\) must be constant.

### Measuring the generated jitter using a digital storage scope

Can measurement prove all this theory? Of course; we found that the measurements match the theory very well. However, there is a catch: How is it possible to see the jitter if the modulating frequency \((\omega_m)\) is significantly lower than the clock frequency? If you were to synchronize the time base with the clock sine wave, a few cycles would not be sufficient to see the whole amount of jitter, especially if the modulating frequency \((\omega_m)\) is much lower than the clock frequency. Increasing the time-base period would add too many cycles on the screen, and the jitter would be difficult to measure. For instance, if the modulating frequency were 100 kHz, and the clock were 125 MHz, the screen would need to display \(1.25 \times 10^6 / 100 \times 10^3 = 1250\) clock cycles. It would be impossible to identify any jitter on the screen.

A technique allows you to measure the jitter even under the conditions mentioned above. Take a look at Equation 5. The integration interval is from \(-\pi/2\) to \(\pi/2\), which corresponds to \(\pi\) radians, or one half of the modulating frequency period. The result suggests that to catch the peak-to-peak jitter generated by the FM signal, you need to look at an interval equal to one half of the \(f_m\) period. Setting the time base to display the whole interval on the screen does not help, but you could set the scope to synchronize on the rising edge of the clock sine wave then start displaying \(1/(2f_m)\) seconds later. Although
the time-base speed is set in such a way that only one or two cycles of the clock would be plotted on the screen (for instance 2 nsec/div for an 8-nsec period), the trigger would be delayed by half the period of the modulating frequency (or the modulation rate). This setting allows the full amount of jitter to be displayed. As a test, you can check that setting the time-base delay to ¼ or ¾ of the modulating signal period, the observed jitter will be less than the amount observed on a ½ setting, which is the actual peak to peak. If you set the time-base delay to one entire modulating frequency period, the jitter on the screen will be zero.

Spectral analysis of the FM signal

It is possible that the amount of jitter the receiver would be able to tolerate is larger than 1 UI—that is, more than the main clock period. Unless specialized equipment is available, a regular digital scope cannot display jitter in excess of 1 UI. If the jitter were larger than 1 UI, the edges would overlap, making it difficult to read the zero crossings’ uncertainty. As an alternative, you could use spectral analysis as a second means of calculating the amount of jitter generated.

Recall from Equation 4 that you can write the frequency-modulated signal as:

\[ x(t) = A_0 \cdot \cos(\omega_0 t + \frac{\Delta \omega}{\omega_m} \cdot \sin(\omega_m t)) = A_0 \cdot \cos(\omega_0 t + \beta \cdot \sin(\omega_m t)) , \]

where \( \beta = \Delta \omega/\omega_m \) is the modulation index.

Now, take a closer look at the frequency content of the FM signal. Using trigonometric identities, \( x(t) \) can be written as:

\[ x(t) = A_0 \cdot \cos(\beta \cdot \sin(\omega_m t)) \cdot \cos(\omega_0 t) - A_0 \cdot \sin(\beta \cdot \sin(\omega_m t)) \cdot \sin(\omega_0 t) . \]

Inspecting Equation 9, you can see that the FM signal consists of a sum of the in phase and the quadrature components of the same clock, whose angular frequency is \( \omega_0 \).

If you further replace

\[ \cos(\beta \cdot \sin(\omega_m t)) = J_0(\beta) + 2 \cdot J_2(\beta) \cdot \cos(2 \cdot \omega_m t) + 2 \cdot J_4(\beta) \cdot \cos(4 \cdot \omega_m t) + ... \]

and

\[ \sin(\beta \cdot \sin(\omega_m t)) = 2 \cdot J_1(\beta) \cdot \sin(\omega_m t) + 2 \cdot J_3(\beta) \cdot \sin(3 \cdot \omega_m t) + ... , \]

then you could rewrite the FM signal as:

\[ x(t) = A_0 \cdot [J_0(\beta) + 2 \cdot J_2(\beta) \cdot \cos(2 \cdot \omega_m t) + ...] \]
\[ - A_0 \cdot [2 \cdot J_1(\beta) \cdot \sin(\omega_m t) + 2 \cdot J_3(\beta) \cdot \sin(3 \cdot \omega_m t) + ...] \]

\[ = A_0 \cdot J_0(\beta) \cdot \cos(\omega_0 t) + \]
\[ + A_0 \cdot J_1(\beta) \cdot [\cos(\omega_0 + \omega_m)t - \cos(\omega_0 - \omega_m)t] \]
\[ + A_0 \cdot J_2(\beta) \cdot [\cos(\omega_0 + 2 \cdot \omega_m)t + \cos(\omega_0 - 2 \cdot \omega_m)t] \]
\[ + A_0 \cdot J_3(\beta) \cdot [\cos(\omega_0 + 3 \cdot \omega_m)t - \cos(\omega_0 - 3 \cdot \omega_m)t] + ... \]

\( J_N(\beta) \) is the dedicated notation for the Bessel function of order \( N \) and variable \( \beta \).

Equation 12 reveals that the spectrum of a frequency-modulated signal consists of a main clock (carrier) \( \omega_0 \) and a series of side components spaced at \( \omega_m \), above and
below the main clock. Their amplitudes are given by $J_N(\beta)$ functions, which depend on both the order of the side band and the modulation index, $\beta$.

**Figure 1** provides a visual representation of the FM sidebands’ distribution: a picture of the well-known Bessel Functions generated from MS Excel spreadsheets.

![Bessel functions](image)

**Figure 1**—Variation of Bessel functions of order 0 to 3 and variable $\beta$ (0 … 15)

It could be shown that the clock ($J_0$) vanishes if $\beta=2.4048$. This observation will help you calibrate the test equipment used for the experiment.

Knowing the spectral distribution of the FM clock displayed on a spectrum analyzer is useful. From the ratio between the sidebands and the main carrier, you could determine the $\beta$ parameter and, further, calculate the amount of jitter even if the latter is higher than 1 UI.
**Experiments**

To confirm the theoretical results shown above, our group performed a few lab experiments. We encourage readers to reproduce these experiments in their own laboratories.

**Figure 2**—The lab setup consists of a Function Generator, whose destination is to externally modulate the RF generator; an RF Generator that generates the frequency modulated clock; an RF splitter used for signal distribution; a Spectrum Analyzer; and a digital oscilloscope.

Before beginning the experiment, carefully calibrate the equipment. Make sure that the frequency deviation set on the RF generator is as accurate as possible, otherwise significant errors may occur. Assuming that the modulation characteristic of the RF generator is linear (the frequency deviation is proportional to the amplitude of the external modulation input signal), calibrate the top end of the modulation scale. An inspection of the Bessel function tables shows that the clock-frequency component (the carrier) cancels itself when the modulation index ($\beta$) equals 2.4048. If the modulating frequency is 100 kHz, the frequency deviation at which you expect the main carrier to cancel is 240.48 kHz. Set the reading on the frequency deviation on the RF generator to 240.48 kHz, then fine-tune the amplitude of the function-generator output to see the carrier cancellation on the spectrum analyzer. The end result of the procedure should look like the picture in **Figure 3**.
The previous step ensures the proper calibration of the frequency deviation at the ends of the deviation range and should be performed every time the modulating frequency changes; recalibration is required to compensate for the frequency dependence of the modulation characteristic of the RF Generator.

The next step is to generate a clean carrier. To achieve this goal, set the frequency deviation of the RF generator to zero. Also, set the output level for easy capture on the oscilloscope and on the spectrum analyzer. The end result of this step should look similar to Figure 4 on the spectrum analyzer and Figures 5 and 6 on the digital oscilloscope. The carrier frequency used for the experiment was set to 125 MHz. The figures show both main and delayed traces on the oscilloscope for reference.
Figure 4—Nonmodulated RF clock on the spectrum analyzer

Figure 5—Nonmodulated clock on the oscilloscope screen; main trace shown
The next step of the experiment applies frequency modulation to the main carrier. The modulating frequency, as provided by the function generator, is 100 kHz, which is also known as the rate at which the carrier’s frequency is moved back and forth. The frequency deviation will be set to 20 kHz from the front panel of the RF generator. Calculate the expected jitter that should occur on the scope display, as well as the ratio between the first sideband and the main carrier as seen on the spectrum analyzer.

Applying Equation 7, the expected peak-to-peak jitter is:

\[
T_j = \frac{2 \cdot \beta}{\pi \cdot f_0} \cdot \frac{\Delta f}{f_m} = \frac{2 \cdot 0.2}{\pi \cdot 100E3} \cdot \frac{20E3}{100E3} = 1.018\text{ns},
\]

and the ratio between the first sideband amplitude and the carrier amplitude is:

\[
R[\text{dB}] = 20 \cdot \log \left( \frac{J_1(\beta)}{J_0(\beta)} \right) = -19.96\text{dB},
\]

where \( \beta = 0.2 \).

Figures 7 and 8 show the measurements taken on the oscilloscope and the spectrum analyzer.
Figure 7—FM clock on the oscilloscope screen; 5-usec-delayed trace shown, $\beta=0.2$

Figure 8—FM clock on the spectrum analyzer, $\beta=0.2$
Choosing the right delay on the oscilloscope is important. As explained above, you need to set the timebase delay for one-half the modulating frequency to capture the maximum deviation of the transition point from the ideal, nonmodulated carrier. Figures 9, 10, and 11 illustrate how setting the timebase delay to $\frac{1}{4}$, $\frac{3}{4}$, or one full modulating-frequency (100-kHz) period affects the measurement.

Figure 9—FM clock on the oscilloscope screen, 2.5-usec delayed trace shown, $\beta = 0.2$
Figure 10—FM clock on the oscilloscope screen, 7.5-usec delayed trace shown, $\beta=0.2$

Figure 11—FM clock on the oscilloscope screen, 10-usec delayed trace shown, $\beta=0.2$
As **Figure 11** shows, it is possible to completely hide the jitter by improperly choosing the delay interval.

To conclude the experiments, we chose a second frequency deviation of 50 kHz, while maintaining the same 100-kHz modulating frequency (or rate). The calculated peak-to-peak jitter and the sideband to main carrier magnitude ratio are:

\[
T_j = 2.55 \text{ns}, \quad \beta \Delta f = 50 \text{kHz}
\]

\[
R(\text{dB}) = -11.76 \text{dB}, \quad \text{where } \beta = 0.5.
\]

**Figures 12** and **13** confirm the calculated results.

**Figure 12**—FM clock on the oscilloscope screen, 5-usec delayed trace shown, \( \beta = 0.5 \)
Figure 13—FM clock on the spectrum analyzer, $\beta=0.5$

Author’s biography

Gabriel Patulea received his MS in electrical engineering from “Politehnica” University in Bucharest, Romania. He has more than 15 years design experience in the analog and RF field. His expertise and interests also include embedded programming and DSP. He currently works for Cisco Systems as a hardware engineer. In his spare time, he enjoys bike riding with his family, building remote-controlled planes, and playing electric guitar.