CHAPTER 9

Frequency Response and Stability of Feedback Amplifiers

9.1 Introduction

In Chapter 8, we considered the effects of negative feedback on circuit parameters such as gain and terminal impedance. We saw that application of negative feedback resulted in a number of performance improvements, such as reduced sensitivity of gain to active-device parameter changes and reduction of distortion due to circuit nonlinearities.

In this chapter, we see the effect of negative feedback on the frequency response of a circuit. The possibility of oscillation in feedback circuits is illustrated, and methods of overcoming these problems by compensation of the circuit are described. Finally, the effect of compensation on the large-signal high-frequency performance of feedback amplifiers is investigated.

Much of the analysis in this chapter is based on the ideal block diagram in Fig. 9.1. This block diagram includes the forward gain $a$ and feedback factor $f$, which are the parameters used in two-port analysis of feedback circuits in Chapter 8. The equations and results in this chapter could be expressed in terms of the parameters used in the return-ratio analysis in Chapter 8 by an appropriate change of variables, as shown in Appendix A9.1.

The equations and relationships in this chapter are general and can be applied to any feedback circuit. However, for simplicity we will often assume the feedback factor $f$ is a positive, unitless constant. One circuit that has such an $f$ is the series-shunt feedback circuit shown in Fig. 8.24. In this circuit, the feedback network is a resistive voltage divider, so $f$ is a constant with $0 \leq f \leq 1$. The forward gain $a$ is a voltage gain that is positive at low frequencies. This circuit gives a noninverting closed-loop voltage gain.

9.2 Relation Between Gain and Bandwidth in Feedback Amplifiers

Chapter 8 showed that the performance improvements produced by negative feedback were obtained at the expense of a reduction in gain by a factor $(1 + T)$, where $T$ is the loop gain. The performance specifications that were improved were also changed by the factor $(1 + T)$.

In addition to the foregoing effects, negative feedback also tends to broadband the amplifier. Consider first a feedback circuit as shown in Fig. 9.1 with a simple basic amplifier whose gain function contains a single pole

$$a(s) = \frac{ab}{1 - \frac{s}{pf}}$$

(9.1)
9.2 Relation Between Gain and Bandwidth in Feedback Amplifiers

where \( a_0 \) is the low-frequency gain of the basic amplifier and \( p_1 \) is the basic-amplifier pole in radians per second. Assume that the feedback path is purely resistive and thus the feedback function \( f \) is a positive constant. Since Fig. 9.1 is an ideal feedback arrangement, the overall gain is

\[
A(s) = \frac{v_o}{v_i} = \frac{a(s)}{1 + a(s)f}
\]  
\hspace{1cm} (9.2)

where the loop gain is \( T(s) = a(s)f \). Substitution of (9.1) in (9.2) gives

\[
A(s) = \frac{a_0}{1 + \frac{f}{p_1}} \cdot \frac{1 + \frac{a_0 f}{p_1}}{1 - \frac{a_0 f}{p_1}} = \frac{a_0}{1 + a_0 f} \cdot \frac{1}{1 - \frac{1}{p_1}} = \frac{a_0}{1 + a_0 f} \cdot \frac{1}{1 - \frac{1}{p_1}}
\]  
\hspace{1cm} (9.3)

From (9.3) the low-frequency gain \( A_0 \) is

\[
A_0 = \frac{a_0}{1 + T_0}
\]  
\hspace{1cm} (9.4)

where

\[
T_0 = a_0 f = \text{low-frequency loop gain}
\]  
\hspace{1cm} (9.5)

The \(-3\,\text{dB} \) bandwidth of the feedback circuit (i.e., the new pole magnitude) is \((1 + a_0 f) \cdot |p_1|\) from (9.3). Thus the feedback has reduced the low-frequency gain by a factor \((1 + T_0)\), which is consistent with the results of Chapter 8, but it is now apparent that the \(-3\,\text{dB} \) frequency of the circuit has been increased by the same quantity \((1 + T_0)\). Note that the gain-bandwidth product is constant. These results are illustrated in the Bode plots of Fig. 9.2, where the magnitudes of
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\[
(1 + T_0) p_1
\]

\( T_0 = 0 \)

\( \sigma \)

\( j \omega \)

\( X \)

\( s \) plane

\[ \text{Figure 9.3} \text{ Locus of the pole of the circuit of Fig. 9.1 as loop gain } T_0 \text{ varies.} \]

\( a(j\omega) \) and \( A(j\omega) \) are plotted versus frequency on log scales. It is apparent that the gain curves for any value of \( T_0 \) are contained in an envelope bounded by the curve of \( |a(j\omega)| \).

Because the use of negative feedback allows the designer to trade gain for bandwidth, negative feedback is widely used as a method for designing broadband amplifiers. The gain reduction that occurs is made up by using additional gain stages, which in general are also feedback amplifiers.

Let us now examine the effect of the feedback on the pole of the overall transfer function \( A(s) \). It is apparent from (9.3) that as the low-frequency loop gain \( T_0 \) is increased, the magnitude of the pole of \( A(s) \) increases. This is illustrated in Fig. 9.3, which shows the locus of the pole of \( A(s) \) in the \( s \) plane as \( T_0 \) varies. The pole starts at \( p_1 \) for \( T_0 = 0 \) and moves out along the negative real axis as \( T_0 \) is made positive. Figure 9.3 is a simple root-locus diagram and will be discussed further in Section 9.5.

9.3 Instability and the Nyquist Criterion

In the above simple example the basic amplifier was assumed to have a single-pole transfer function, and this situation is closely approximated in practice by internally compensated general-purpose op amps. However, many amplifiers have multipole transfer functions that cause deviations from the above results. The process of compensation overcomes these problems, as will be seen later.

Consider an amplifier with a three-pole transfer function

\[
a(s) = \left( 1 - \frac{a_0}{p_1} \right) \left( 1 - \frac{a_0}{p_2} \right) \left( 1 - \frac{a_0}{p_3} \right)
\]

(9.6)

where \( |p_1|, |p_2|, \) and \( |p_3| \) are the pole magnitudes in rad/s. The poles are shown in the \( s \) plane in Fig. 9.4 and gain magnitude \( |a(j\omega)| \) and phase \( \text{ph } a(j\omega) \) are plotted versus frequency in Fig. 9.5 assuming about a factor of 10 separation between the poles. Only asymptotes are

\[ \text{Figure 9.4} \text{ Poles of an amplifier in the } s \text{ plane.} \]
9.3 Instability and the Nyquist Criterion

Figure 9.5 Gain and phase versus frequency for a circuit with a three-pole transfer function.

shown for the magnitude plot. At frequencies above the first pole magnitude \( |p_1| \), the plot of \( \log_{10} a \) falls at 6 dB/octave and \( \text{ph} \) \( a(j\omega) \) approaches \(-90^\circ\). Above \( |p_2| \) these become 12 dB/octave and \(-180^\circ\), and above \( |p_3| \) they become 18 dB/octave and \(-270^\circ\). The frequency where \( \text{ph} \) \( a(j\omega) = -180^\circ \) has special significance and is marked \( \omega_{180} \), and the value of \( |a(j\omega)| \) at this frequency is \( a_{180} \). If the three poles are fairly widely separated (by a factor of 10 or more), the phase shifts at frequencies \( |p_1|, |p_2|, \) and \( |p_3| \) are approximately \(-45^\circ\), \(-135^\circ\), and \(-225^\circ\), respectively. This will now be assumed for simplicity. In addition, the gain magnitude will be assumed to follow the asymptotic curve and the effect of these assumptions in practical cases will be considered later.

Now consider this amplifier connected in a feedback loop as in Fig. 9.1 with \( f \) a positive constant. Since \( f \) is constant, the loop gain \( T(j\omega) = a(j\omega)f \) will have the same variation with frequency as \( a(j\omega) \). A plot of \( a(j\omega)f \) in magnitude and phase on a polar plot (with \( \omega \) as a parameter) can thus be drawn using the data of Fig. 9.5 and the magnitude of \( f \). Such a plot for this example is shown in Fig. 9.6 (not to scale) and is called a Nyquist diagram. The variable on the curve is frequency and varies from \( \omega = -\infty \) to \( \omega = \infty \). For \( \omega = 0 \), \( |T(j\omega)| = T_0 \) and \( \text{ph} \) \( T(j\omega) = 0 \), and the curve meets the real axis with an intercept \( T_0 \). As \( \omega \) increases, as Fig. 9.5 shows, \( |a(j\omega)| \) decreases and \( \text{ph} \) \( a(j\omega) \) becomes negative and thus the plot is in the fourth quadrant. As \( \omega \to \infty \), \( \text{ph} \) \( a(j\omega) \to -270^\circ \) and \( |a(j\omega)| \to 0 \). Consequently, the plot is asymptotic to the origin and is tangent to the imaginary axis. At the frequency \( \omega_{180} \) the phase is \(-180^\circ\) and the curve crosses the negative real axis. If \( |a(j\omega)T_0| > 1 \) at this point, the Nyquist diagram will encircle the point \((-1, 0)\) as shown, and this has particular significance, as will
now become apparent. For the purposes of this treatment, the Nyquist criterion for stability of the amplifier can be stated as follows:

"Consider a feedback amplifier with a stable \( T(s) \) (i.e., all poles of \( T(s) \) are in the left half-plane). If the Nyquist plot of \( T(j\omega) \) encircles the point \((-1, 0)\), the feedback amplifier is unstable."

This criterion simply amounts to a mathematical test for poles of transfer function \( A(s) \) in the right half-plane. If the Nyquist plot encircles the point \((-1, 0)\), the amplifier has poles in the right half-plane and the circuit will oscillate. In fact the number of encirclements of the point \((-1, 0)\) gives the number of right half-plane poles and in this example there are two. The significance of poles in the right half-plane can be seen by assuming that a circuit has a pair of complex poles at \((\sigma_1 \pm j\omega_1)\) where \(\sigma_1\) is positive. The transient response of the circuit then contains a term \(K_1 \exp(\sigma_1 t) \sin(\omega_1 t)\), which represents a growing sinusoid if \(\sigma_1\) is positive. \((K_1\) is a constant representing initial conditions.) This term is then present even if no further input is applied, and a circuit behaving in this way is said to be unstable or oscillatory.

The significance of the point \((-1, 0)\) can be appreciated if the Nyquist diagram is assumed to pass through this point. Then at the frequency \(\omega_{180}\), \(T(j\omega) = a(j\omega) f = -1\) and \(A(j\omega) = \infty\) using (9.2) in the frequency domain. The feedback amplifier is thus calculated to have a forward gain of infinity, and this indicates the onset of instability and oscillation. This situation corresponds to poles of \(A(s)\) on the \(j\omega\) axis in the \(s\) plane. If \(T_0\) is then increased by increasing \(a_0\) or \(f\), the Nyquist diagram expands linearly and then encircles \((-1, 0)\). This corresponds to poles of \(A(s)\) in the right half-plane, as shown in Fig. 9.7.

From the above criterion for stability, a simpler test can be derived that is useful in most common cases.

"If \(|T(j\omega)| > 1\) at the frequency where \(\text{ph} T(j\omega) = -180^\circ\), then the amplifier is unstable."

The validity of this criterion for the example considered here is apparent from inspection of Fig. 9.6 and application of the Nyquist criterion.

In order to examine the effect of feedback on the stability of an amplifier, consider the three-pole amplifier with gain function given by (9.6) to be placed in a negative-feedback loop with \(f\) constant. The gain (in decibels) and phase of the amplifier are shown again in Fig. 9.8, and also plotted is the quantity 20 \(\log_{10} f\). The value of 20 \(\log_{10} f\) is approximately equal
9.3 Instability and the Nyquist Criterion

Figure 9.7 Pole positions corresponding to different Nyquist diagrams.

Figure 9.8 Amplifier gain and phase versus frequency showing the phase margin.

to the low-frequency gain in decibels with feedback applied since

\[
A_0 = \frac{a_0}{1 + a_0 f}
\]  

and thus

\[
\frac{1}{f} = A_0
\]
Consider the vertical distance between the curve of $20 \log_{10} |A(j\omega)|$ and the line $20 \log_{10} 1/f$ in Fig. 9.8. Since the vertical scale is in decibels this quantity is
\[
x = 20 \log_{10} |A(j\omega)| - 20 \log_{10} 1/f \tag{9.9}
\]
\[
= 20 \log_{10} \frac{|A(j\omega)|}{|A(j\omega)|} \tag{9.10}
\]
Thus the distance $x$ is a direct measure in decibels of the loop-gain magnitude, $|T(j\omega)|$. The point where the curve of $20 \log_{10} |A(j\omega)|$ intersects the line $20 \log_{10} 1/f$ is the point where the loop-gain magnitude $|T(j\omega)|$ is 0 dB or unity, and the curve of $|A(j\omega)|$ in decibels in Fig. 9.8 can thus be considered a curve of $|T(j\omega)|$ in decibels if the dotted line at $20 \log_{10} 1/f$ is taken as the new zero axis.

The simple example of Section 9.1 showed that the gain curve versus frequency with feedback applied ($20 \log_{10} |A(j\omega)|$) follows the $20 \log_{10} A_0$ line until it intersects the gain curve $20 \log_{10} |A(j\omega)|$. At higher frequencies the curve $20 \log_{10} |A(j\omega)|$ simply follows the curve of $20 \log_{10} |A(j\omega)|$ for the basic amplifier. The reason for this is now apparent in that at the higher frequencies the loop gain $|T(j\omega)| \to 0$ and the feedback then has no influence on the gain of the amplifier.

Figure 9.8 shows that the loop-gain magnitude $|T(j\omega)|$ is unity at frequency $\omega_0$. At this frequency the phase of $T(j\omega)$ has not reached $-180^\circ$ for the case shown, and using the modified Nyquist criterion stated above we conclude that this feedback loop is stable. Obviously $|T(j\omega)| < 1$ at the frequency where $\text{ph } T(j\omega) = -180^\circ$. If the polar Nyquist diagram is sketched for this example, it does not encircle the point $(−1, 0)$.

As $|T(j\omega)|$ is made closer to unity at the frequency where $\text{ph } T(j\omega) = -180^\circ$, the amplifier has a smaller margin of stability, and this can be specified in two ways. The most common is the phase margin, which is defined as follows:

Phase margin $= 180^\circ + (\text{ph } T(j\omega))$ at frequency where $|T(j\omega)| = 1$. The phase margin is indicated in Fig. 9.8 and must be greater than $0^\circ$ for stability.

Another measure of stability is the gain margin. This is defined to be $1/|T(j\omega)|$ in decibels at the frequency where $\text{ph } T(j\omega) = -180^\circ$, and this must be greater than 0 dB for stability.

The significance of the phase-margin magnitude is now explored. For the feedback amplifier considered in Section 9.1, where the basic amplifier has a single-pole response, the phase margin is obviously $90^\circ$ if the low-frequency loop gain is reasonably large. This is illustrated in Fig. 9.9 and results in a very stable amplifier. A typical lower allowable limit for the phase margin in practice is $45^\circ$, with a value of $60^\circ$ being more common.

Consider a feedback amplifier with a phase margin of $45^\circ$ and a feedback function $f$ that is real (and thus constant). Then
\[
\text{ph } T(j\omega) = -135^\circ \tag{9.11}
\]
where $\omega_0$ is the frequency defined by
\[
|T(j\omega)| = 1 \tag{9.12}
\]
Now $|T(j\omega)| = |A(j\omega)|f$ implies that
\[
|A(j\omega)| = \frac{1}{f} \tag{9.13}
\]
assuming that $f$ is positive real.
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The overall gain is

\[ A(j\omega) = \frac{a(j\omega)}{1 + T(j\omega)} \]  

Substitution of (9.11) and (9.12) in (9.14) gives

\[ A(j\omega_0) = \frac{a(j\omega_0)}{1 + e^{-j135^\circ}} = \frac{a(j\omega_0)}{0.3 - 0.7j} \]

and thus

\[ |A(j\omega_0)| = \frac{|a(j\omega_0)|}{0.76} = \frac{1.3}{f} \]  

using (9.13).

The frequency \( \omega_0 \), where \(|T(j\omega_0)| = 1\), is the nominal \( -3 \)-dB point for a single-pole basic amplifier, but in this case there is 2.4 dB \((1.3 \times)\) of peaking above the low-frequency gain of \(1/f\).

Consider a phase margin of \( 60^\circ \). At the frequency \( \omega_0 \) in this case

\[ \text{ph} T(j\omega_0) = -120^\circ \]  

and

\[ |T(j\omega_0)| = 1 \]  

Following a similar analysis we obtain

\[ |A(j\omega_0)| = \frac{1}{f} \]

In this case there is no peaking at \( \omega = \omega_0 \), but there has also been no gain reduction at this frequency.

Finally, the case where the phase margin is \( 90^\circ \) can be similarly calculated. In this case

\[ \text{ph} T(j\omega_0) = -90^\circ \]
A similar analysis gives

\[ |A(j\omega_0)| = \frac{0.7}{\mathcal{F}} \]  

As expected in this case, the gain at frequency \( \omega_0 \) is 3 dB below the midband value.

These results are illustrated in Fig. 9.10, where the normalized overall gain versus frequency is shown for various phase margins. The plots are drawn assuming the response is dominated by the first two poles of the transfer function, except for the case of the 90° phase margin, which has one pole only. As the phase margin diminishes, the gain peak becomes larger until the gain approaches infinity and oscillation occurs for phase margin = 0°. The gain peak usually occurs close to the frequency where \( |T(j\omega)| = 1 \), but for a phase margin of 60° there is 0.2 dB of peaking just below this frequency. Note that after the peak, the gain curves approach an asymptote of \(-12\) dB/octave for phase margins other than 90°. This is because the open-loop gain falls at \(-12\) dB/octave due to the presence of two poles in the transfer function.

The simple tests for stability of a feedback amplifier (i.e., positive phase and gain margins) can only be applied when the phase and gain margins are uniquely defined. The phase margin is uniquely defined if there is only one frequency at which the magnitude of the loop gain equals one. Similarly, the gain margin is uniquely defined if there is only one frequency at which the phase of the loop gain equals \(-180°\). In most feedback circuits, these margins are uniquely defined. However, if either of these margins is not uniquely defined, then stability should be checked using a Nyquist diagram and the Nyquist criterion.
The loop gain $T = af$ can be examined to determine the stability of a feedback circuit, as explained in this section. Alternatively, these measures of stability can be applied to the return ratio $\beta$, as explained in Appendix A9.1. Techniques for simulating $\beta$ and $T = af$ using SPICE have been developed, based on methods for measuring loop transmission. These techniques measure the loop transmission at the closed-loop dc operating point. An advantage of SPICE simulation of the loop transmission is that parasitics that might have an important effect are included. For example, parasitic capacitance at the op-amp input introduces frequency dependence in the feedback network in Fig. 8.24, which may degrade the phase margin.

9.4 Compensation

9.4.1 Theory of Compensation

Consider again the amplifier whose gain and phase is shown in Fig. 9.8. For the feedback circuit in which this was assumed to be connected, the forward gain was $A_0$, as shown in Fig. 9.8, and the phase margin was positive. Thus, the circuit was stable. It is apparent, however, that if the amount of feedback is increased by making $f$ larger (and thus $A_0$ smaller), oscillation will eventually occur. This is shown in Fig. 9.11, where $f_1$ is chosen to give a zero phase margin and the corresponding overall gain is $A_1 \approx 1/f_1$. If the feedback is increased to $f_2$ (and $A_2 \approx 1/f_2$ is the overall gain), the phase margin is negative and the circuit will oscillate. Thus, if this amplifier is to be used in a feedback loop with loop gain larger than $a_0 f_1$, efforts

![Figure 9.11 Gain and phase versus frequency for a three-pole basic amplifier. Feedback factor $f_1$ gives a zero phase margin and factor $f_2$ gives a negative phase margin.](image)

$20 \log_{10} \approx 20 \log_{10} A_1 + 20 \log_{10} f_1$

$20 \log_{10} \approx 20 \log_{10} A_2 + 20 \log_{10} f_2$

$\omega$ log scale

$|\Phi(\omega)|$

$1/f_1$

$1/f_2$

$P_{\text{in}}(\omega)$
must be made to increase the phase margin. This process is known as compensation. Note that without compensation, the forward gain of the feedback amplifier cannot be made less than \( A_1 \approx 1/f \) because of the oscillation problem.

The simplest and most common method of compensation is to reduce the bandwidth of the amplifier (often called narrowbanding). That is, a dominant pole is deliberately introduced into the amplifier to force the phase shift to be less than \(-180^\circ\) when the loop gain is unity. This involves a direct sacrifice of the frequency capability of the amplifier.

If \( f \) is constant, the most difficult case to compensate is \( f = 1 \), which is a unity-gain feedback configuration. In this case the loop-gain curve is identical to the gain curve of the basic amplifier. Consider this situation and assume that the basic amplifier has the same characteristic as in Fig. 9.11. To compensate the amplifier, we introduce a new dominant pole with magnitude \(|p_D|\), as shown in Fig. 9.12, and assume that this does not affect the original amplifier poles with magnitudes \(|p_1|\), \(|p_2|\), and \(|p_3|\). This is often not the case but is assumed here for purposes of illustration.

The introduction of the dominant pole with magnitude \(|p_D|\) into the amplifier gain function causes the gain magnitude to decrease at 6 dB/octave until frequency \(|p_1|\) is reached, and over this region the amplifier phase shift asymptotes to \(-90^\circ\). If frequency \(|p_D|\) is chosen so that the gain \( |a_j(\omega)| \) is unity at frequency \(|p_1|\) as shown, then the loop gain is also unity at frequency \(|p_1|\) for the assumed case of unity feedback with \( f = 1 \). The phase margin in this case is then \(45^\circ\), which means that the amplifier is stable. The original amplifier would have been unstable in such a feedback connection.

![Figure 9.12](image-url) 
*Figure 9.12* Gain and phase versus frequency for a three-pole basic amplifier. Compensation for unity-gain feedback operation (\( f = 1 \)) is achieved by introduction of a negative real pole with magnitude \(|p_D|\).*
The price that has been paid for achieving stability in this case is that with the feedback removed, the basic amplifier has a unity-gain bandwidth of only \( |p_1| \), which is much less than before. Also, with feedback applied, the loop gain now begins to decrease at a frequency \( |p_2| \), and all the benefits of feedback diminish as the loop gain decreases. For example, in Chapter 8 it was shown that shunt feedback at the input or output of an amplifier reduces the basic terminal impedance by \( (1 + |T(\omega)|) \). Since \( T(\omega) \) is frequency dependent, the terminal impedance of a shunt-feedback amplifier will begin to rise when \( |T(\omega)| \) begins to decrease. Thus the high-frequency terminal impedance will appear inductive, as in the case of \( z_0 \) for an emitter follower, which was calculated in Chapter 7. (See Problem 9.8.)

**EXAMPLE**

Calculate the dominant-pole magnitude required to give unity-gain compensation of the 702 op amp with a phase margin of 45°. The low frequency gain is \( a_0 = 3600 \) and the circuit has poles at \( -|p_1|/2\pi = 1 \) MHz, \( -|p_2|/2\pi = 4 \) MHz, and \( -|p_3|/2\pi = 40 \) MHz.

In this example, the second pole \( p_2 \) is sufficiently close to \( p_1 \) to produce significant phase shift at the amplifier \( 3 \)-dB frequency. The approach to this problem will be to use the approximate results developed above to obtain an initial estimate of the required dominant-pole magnitude and then to empirically adjust this estimate to obtain the required results.

The results of Fig. 9.12 indicate that a dominant pole with magnitude \( |p_{D}| \) should be introduced so that gain \( a_0 = 3600 \) is reduced to unity at \( |p_1|/2\pi = 1 \) MHz with a 6-dB/octave decrease as a function of frequency. The product \(|\omega_j|\) is constant where the slope of the gain-magnitude plot is \(-6\) dB/octave; therefore

\[
\frac{|p_{D}|}{2\pi} = \frac{1}{|a_0|} \frac{|p_1|}{2\pi} = \left( \frac{6}{3600} \right) \times 2\pi = 278 \text{ Hz}
\]

This would give a transfer function

\[
a(j\omega) = \left( 1 + \frac{j\omega}{|p_{D}|} \right) \left( 1 + \frac{j\omega}{|p_1|} \right) \left( 1 + \frac{j\omega}{|p_2|} \right) \left( 1 + \frac{j\omega}{|p_3|} \right)
\]

where the pole magnitudes are in radians per second. Equation 9.21 gives a unity-gain frequency \(|\omega_j|\) of 780 kHz. This is slightly below the design value of 1 MHz because the actual gain curve is 3 dB below the asymptote at the break frequency \( |p_1| \). At 780 kHz the phase shift obtained from (9.21) is \(-135^\circ\) instead of the desired \(-135^\circ\) and this includes a contribution of \(-11^\circ\) from pole \( p_2 \). Although this result is close enough for most purposes, a phase margin of precisely 45° can be achieved by empirically reducing \(|p_2|\) until (9.21) gives a phase shift of \(-135^\circ\) at the unity gain frequency. This occurs for \(|p_{D}|/2\pi = 260 \) Hz, which gives a unity-gain frequency of 780 kHz.

Consider now the performance of the amplifier whose characteristic is shown in Fig. 9.12 (with dominant pole magnitude \(|p_{D}|\)) when used in a feedback loop with \( f < 1 \) (i.e., overall gain \( A_0 > 1 \)). This case is shown in Fig. 9.13. The loop gain now falls to unity at frequency \( \omega_0 \) and the phase margin of the circuit is now approximately 90°. The \(-3\)-dB bandwidth of the feedback circuit is \( \omega_0 \). The circuit now has more compensation than is needed, and, in fact, bandwidth is being wasted. Thus, although it is convenient to compensate an amplifier for unity gain and then use it unchanged for other applications (as is done in many op amps), this procedure is quite wasteful of bandwidth. Fixed-gain amplifiers that are designed for applications where maximum bandwidth is required are usually compensated for a specified phase margin (typically 45° to 60°) at the required gain value. However, op amps are
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20 log10 \( a_0 \) \( | \frac{a(j \omega)}{p_1} | \) dB

\( \omega = |p_1|/A_0 \) if unity-gain compensation had been used. Obviously, since \( A_0 \) can be large, the improvement in bandwidth is significant.

In the compensation schemes discussed above, an additional dominant pole was assumed to be added to the amplifier, and the original amplifier poles were assumed to be unaffected by this procedure. In terms of circuit bandwidth, a much more efficient way to compensate the amplifier is to add capacitance to the circuit in such a way that the original amplifier dominant pole magnitude \( |p_1| \) is reduced so that it performs the compensation function. This technique requires access to the internal nodes of the amplifier, and knowledge of the nodes in the circuit where added capacitance will reduce frequency \( |p_1| \).

Consider the effect of compensating for unity-gain operation the amplifier characteristic of Fig. 9.11 in this way. Again assume that higher frequency poles \( p_2 \) and \( p_3 \) are unaffected by this procedure. In fact, depending on the method of compensation, these poles are usually moved up or down in magnitude by the compensation. This point will be taken up later.

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Figure 9.13 Gain and phase versus frequency for an amplifier compensated for use in a feedback loop with \( f = 1 \) and a phase margin of 45°. The phase margin is shown for operation in a feedback loop with \( f < 1 \).

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general-purpose circuits that are used with differing feedback networks with \( f \) values ranging from 0 to 1. Optimum bandwidth is achieved in such circuits if the compensation is tailored to the gain value required, and this approach gives much higher bandwidths for high gain values, as seen in Fig. 9.14. This figure shows compensation of the amplifier characteristic of Fig. 9.11 for operation in a feedback circuit with forward gain \( A_0 \). A dominant pole is added with magnitude \( |p'_D| \) to give a phase margin of 45°. Frequency \( |p'_D| \) is obviously \( \gg |p_D| \), and the \( -3 \) dB bandwidth of the feedback amplifier is nominally \( |p_1| \), at which frequency the loop gain is 0 dB (disregarding peaking). The \( -3 \) dB frequency from Fig. 9.13 would be only \( \omega_c = |p_1|/A_0 \) if unity-gain compensation had been used. Obviously, since \( A_0 \) can be large, the improvement in bandwidth is significant.
9.4 Compensation

Gain and phase versus frequency for an amplifier compensated for use in a feedback loop with \( f < 1 \) and a phase margin of 45°. Compensation is achieved by adding a new pole \( p'_D \) to the amplifier.

Compensation of the amplifier by reducing \( |p_1| \) is shown in Fig. 9.15. For a 45° phase margin in a unity-gain feedback configuration, dominant pole magnitude \( |p'_1| \) must cause the gain to fall to unity at frequency \( |p_2| \) (the second pole magnitude). Thus the nominal bandwidth in a unity-gain configuration is \( |p_2| \), and the loop gain is unity at this frequency. This result can be contrasted with a bandwidth of \( |p_1| \) as shown in Fig. 9.12 for compensation achieved by adding another pole with magnitude \( |p_D| \) to the amplifier. In practical amplifiers, frequency \( |p_2| \) is often 5 or 10 times frequency \( |p_1| \) and substantial improvements in bandwidth are thus achieved.

The results of this section illustrate why the basic amplifier of a feedback circuit is usually designed with as few stages as possible. Each stage of gain inevitably adds more poles to the transfer function, complicating the compensation problem, particularly if a wide bandwidth is required.

9.4.2 Methods of Compensation

In order to compensate a circuit by the common method of narrowbanding described above, it is necessary to add capacitance to create a dominant pole with the desired magnitude. One method of achieving this is shown in Fig. 9.16, which is a schematic of the first two stages of a simple amplifier. A large capacitor \( C \) is connected between the collectors of the input stage. The output stage, which is assumed relatively broadband, is not shown. A differential half-circuit of Fig. 9.16 is shown in Fig. 9.17, and it should be noted that the compensation
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Figure 9.15 Gain and phase versus frequency for an amplifier compensated for use in a feedback loop with $f = 1$ and a phase margin of 45°. Compensation is achieved by reducing the magnitude $|p_1|$ of the dominant pole of the original amplifier.

Figure 9.16 Compensation of an amplifier by introduction of a large capacitor $C$. The capacitor is doubled in the half-circuit. The major contributions to the dominant pole of a circuit of this type (if $R_S$ is not large) come from the input capacitance of $Q_4$ and Miller capacitance associated with $Q_4$. Thus the compensation as shown will reduce the magnitude of the dominant pole of the original amplifier so that it performs the required compensation function. Almost certainly, however, the higher frequency poles of the amplifier will also be changed by the addition of $C$. In practice, the best method of approaching the compensation
design is to use computer simulation to determine the original pole positions. A first estimate of 
C is made on the assumption that the higher frequency poles do not change in magnitude and a 
new computer simulation is made with C included to check this assumption. Another estimate 
of C is then made on the basis of the new simulation, and this process usually converges after 
several iterations.

The magnitude of the dominant pole of Fig. 9.17 can be estimated using zero-value time 
constant analysis. However, if the value of C required is very large, this capacitor will dominate 
and a good estimate of the dominant pole can be made by considering C only and ignoring 
other circuit capacitance. In that case the dominant-pole magnitude is

$$ |\mu_D| = \frac{1}{2CR} $$  \hfill (9.22)

where

$$ R = R_{L1} || R_d $$  \hfill (9.23)

and

$$ R_d = r_{ds} + r_{es} $$  \hfill (9.24)

One disadvantage of the above method of compensation is that the value of C required is quite 
large (typically > 1000 pF) and cannot be realized on a monolithic chip.

Many general-purpose op amps have unity-gain compensation included on the monolithic 
chip and require no further compensation from the user. (The sacrifice in bandwidth caused by 
this technique when using gain other than unity was described earlier.) In order to realize an 
internally compensated monolithic op amp, compensation must be achieved using capacitance 
less than about 50 pF. This can be achieved using Miller multiplication of the capacitance as 
in the 741 op amp, which uses a 30 pF compensation capacitor and was analyzed in previous 
editions of this book.

As well as allowing use of a small capacitor that can be integrated on the monolithic chip, 
this type of compensation has another significant advantage. This is due to the phenomenon 
of pole splitting,$^8$ in which the dominant pole moves to a lower frequency while the next 
pole moves to a higher frequency. The splitting of the two low-frequency poles in practical 
op amps is often a rather complex process involving other higher frequency poles and zeros 
as well. However, the process involved can be illustrated with the two-stage op-amp model in 
Fig. 9.18. The input is from from a current $i_v$, which stems from the transconductance of the first 
stage times the op-amp differential input voltage. Resistors $R_1$ and $R_2$ represent the total shunt 
resistances at the output of the first and second stages, including transistor input and output 
resistances. Similarly, $C_1$ and $C_2$ represent the total shunt capacitances at the same places. 
Capacitor C represents transistor collector-base capacitance of the amplifying transistor in the 
second stage plus the compensation capacitance.
For the circuit of Fig. 9.18, 
\[ i_i = \frac{v_1}{R_1} + v_1 C_1 s + (v_1 - v_o) C_s \]  
(9.25)  
\[ g_m v_1 + \frac{v_o}{R_2} + v_o C_2 s + (v_o - v_1) C_s = 0 \]  
(9.26)  
From (9.25) and (9.26) 
\[ \frac{v_o}{i_i} = \frac{(g_m - C_s) R_2 R_1}{1 + s[C_2 + C] R_2 + [C_1 + C] R_1 + g_m R_2 R_1 (C + s^2 R_2 R_1 C_2 C_1 + C C_2 + C C_1)} \]  
(9.27)  
The circuit transfer function has a positive real zero at 
\[ z = \frac{g_m}{C} \]  
(9.27a)  
which usually has such a large magnitude in bipolar circuits that it can be neglected. This is often not the case in MOS circuits because of their lower \( g_m \). This point is taken up later.

The circuit has a two-pole transfer function. If \( p_1 \) and \( p_2 \) are the poles of the circuit, then the denominator of (9.27) can be written 
\[ D(s) = \left( 1 - \frac{s}{p_1} \right) \left( 1 - \frac{s}{p_2} \right) \]  
(9.28)  
and thus 
\[ D(s) \approx 1 - \frac{s}{p_1} + \frac{s^2}{p_1 p_2} \]  
(9.29)  
if the poles are real and widely separated, which is usually true. Note that \( p_1 \) is assumed to be the dominant pole.

If the coefficients in (9.27) and (9.30) are equated then 
\[ p_1 = \frac{1}{(C_2 + C) R_2 + (C_1 + C) R_1 + g_m R_2 R_1 C} \]  
(9.31)  
and this can be approximated by 
\[ p_1 \approx -\frac{1}{g_m R_2 R_1 C} \]  
(9.32)
since the Miller effect due to $C$ will be dominant if $C$ is large and $g_mR_1, g_mR_2 \gg 1$. Equation 9.31 is the same result for the dominant pole as is obtained using zero-value time constant analysis.

The nondominant pole $p_2$ can now be estimated by equating coefficients of $s^2$ in (9.27) and (9.30) and using (9.32).

$$p_2 \approx -\frac{g_mC}{C_2C_1 + C(C_2 + C_1)} \quad (9.33)$$

Equation 9.32 indicates that the dominant-pole magnitude $|p_1|$ decreases as $C$ increases, whereas (9.33) shows that $|p_2|$ increases as $C$ increases. Thus, increasing $C$ causes the poles to split apart. The dominant pole moves to a lower frequency because increasing $C$ increases the time constant associated with the output node of the first stage in Fig. 9.18. The reason the nondominant pole moves to a higher frequency is explained below.

Equation 9.33 can be interpreted physically by associating $p_2$ with the output node in Fig. 9.18. Then

$$p_2 = -\frac{1}{R_oC_T} \quad (9.33a)$$

where $R_o$, is the output resistance including negative feedback around the second stage through $C$, and $C_T$ is the total capacitance from the output node to ground. The output resistance is

$$R_o = \frac{R_2}{1 + T} \quad (9.33b)$$

where $R_2$ is the open-loop output resistance, and $T$ is the loop gain around the second stage through capacitor $C$, which is the open-loop gain, $g_mR_2$, times the feedback factor, $f$. Therefore,

$$R_o = \frac{R_2}{1 + g_mR_2f} \approx \frac{1}{g_mf} \quad (9.33c)$$

assuming that $T = g_mR_2f \gg 1$. Since $p_2$ is a high frequency, we will find $f$ at high frequency $\omega_0$, where $1/\omega_0C_1 \ll R_1$. Then the feedback around the second stage is controlled by a capacitive voltage divider and

$$f \approx \frac{C}{C + C_1} \quad (9.33d)$$

Thus,

$$R_o \approx C + C_1 \quad (9.33e)$$

The total capacitance from the output node to ground is $C_T$ in parallel with the series combination of $C$ and $C_1$:

$$C_T = C_2 + \frac{CC_1}{C + C_1} = \frac{CC_2 + C_1C_2 + CC_1}{C + C_1} \quad (9.33f)$$

Substituting (9.33e) and (9.33f) into (9.33a) gives (9.33).

Equations 9.33d and 9.33f show that increasing $C$ increases the feedback factor but has little effect on the total capacitance in shunt with the output node because $C$ is in series with $C_1$. As a result, increasing $C$ reduces the output resistance and increases the frequency of the nondominant pole. In the limit as $C \to \infty$, the feedback factor approaches unity, and $p_2 \to -g_m/(C_2 + C_1)$. In practice, however, (9.33d) shows that the feedback factor is less than unity, which limits the increase in the magnitude of the nondominant pole frequency.
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Poles split

$\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} - \frac{1}{R_1 C_1} \times \frac{1}{R_2 C_2} - \frac{j \omega}{s}$

Figure 9.19  Locus of the poles of the circuit of Fig. 9.18 as $C$ is increased from zero, for the case $-1/(R_1 C_1) > -1/(R_2 C_2).$

On the other hand, with $C = 0$, the poles of the circuit of Fig. 9.18 are

\[ p_1 = -\frac{1}{R_1 C_1} \] (9.34a)

\[ p_2 = -\frac{1}{R_2 (C_2 + C)} \] (9.34b)

Thus as $C$ increases from zero, the locus of the poles of the circuit of Fig. 9.18 is as shown in Fig. 9.19.

Another explanation of pole splitting is as follows. The circuit in Fig. 9.18 has two poles. The compensation capacitor across the second stage provides feedback and causes the second stage to act like an integrator. The two poles split apart as $C$ increases. One pole moves to a low frequency (toward dc), and the other moves to a high frequency (toward $-\infty$) to approximate an ideal integrator, which has only one pole at dc.

The previous calculations have shown how compensation of an amplifier by addition of a large Miller capacitance to a single transistor stage causes the nondominant pole to move to a much higher frequency. For the sake of comparison, consider compensating the circuit in Fig. 9.18 without adding capacitance to $C$ by making $C_1$ large enough to produce a dominant pole. Then the pole can be calculated from (9.31) as $p_1 \approx -1/R_1 C_1$. The nondominant pole can be estimated by equating coefficients of $s^2$ in (9.27) and (9.30) and using this value of $p_1$. This gives $p_2 \approx -1/R_2 (C_2 + C)$. This value of $p_2$ is approximately the same as that given by (9.34b), which is for $C = 0$ and is before pole splitting occurs. Thus, creation of a dominant pole in the circuit of Fig. 9.18 by making $C_1$ large will result in a second pole magnitude $|p_2|$ that is much smaller than that obtained if the dominant pole is created by increasing $C$. As a consequence, the realizable bandwidth of the circuit when compensated in this way is much smaller than that obtained with Miller-effect compensation. Also, without using the Miller effect, the required compensation capacitor often would be too large to be included on a monolithic chip. The same general conclusions are true in the more complex situation that exists in many practical op amps.

The results derived in this section are useful in further illuminating the considerations of Section 7.3.3. In that section, it was stated that in a common-source cascade, the existence of drain-gate capacitance tends to cause pole splitting and to produce a dominant-pole situation. If the equivalent circuit of Fig. 9.18 is taken as a representative section of a cascade of common-source stages ($C_2$ is the input capacitance of the following stage) and capacitor $C$ is taken as $C_{gd}$, the calculations of this section show that the presence of $C_{gd}$ does, in fact, tend to produce a dominant-pole situation because of the pole splitting that occurs. Thus, the zero-value time constant approach gives a good estimate of $\omega_{3dB}$ in such circuits.

The theory of compensation that was developed in this chapter was illustrated with some bipolar-transistor circuit examples. The theory applies in general to any active circuit, but the unique device parameters of MOSFETs cause some of the approximations that were made in
the preceding analyses to become invalid. The special aspects of MOS amplifier compensation are now considered.

9.4.3 Two-Stage MOS Amplifier Compensation

The basic two-stage CMOS op amp topology shown in Fig. 6.16 is essentially identical to its bipolar counterpart. As a consequence, the equivalent circuit of Fig. 9.18 can be used to represent the second stage with its compensation capacitance. The poles of the circuit are again given by (9.32) and (9.33) and the zero by (9.27a). In the case of the MOS transistor, however, the value of $g_m$ is typically an order of magnitude lower than for a bipolar transistor, and the break frequency caused by the right half-plane zero in (9.27) may actually fall below the nominal unity-gain frequency of the amplifier. The effect of this is shown in Fig. 9.20. At the frequency $|z|$ the gain characteristic of the amplifier flattens out because of the contribution to the gain of $+6$ dB/octave from the zero. In the same region the phase is made $90^\circ$ more negative by the positive real zero. As a consequence, the amplifier will have negative phase margin and be unstable when the influence of the next most dominant pole is felt. In effect, the zero halts the gain roll-off intended to stabilize the amplifier and simultaneously pushes the phase in the negative direction. Note also from (9.33) that the low $g_m$ of the MOSFET will tend to reduce the value of $|p_2|$ relative to a bipolar amplifier.

Another way to view this problem is to note from Fig. 9.18 that at high frequencies, feedforward through $C$ tends to overwhelm the normal gain path via $g_m$ of the second stage.

![Figure 9.20](image-url) Typical gain and phase of the CMOS op amp of Fig. 6.16.