Windowing High-Resolution ADC Data – Part 2

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Abstract

Analyzing data from ADCs requires the use of windowing functions for spectral estimation and analysis but different windows suit different purposes. National Semiconductor’s new WaveVision 5 software provides a family of mathematically simple windowing functions that spans a fundamental tradeoff and allows the flexibility to meet a wide range of user applications. Part 2 of this article presents the Cosine-Sum family of windowing functions available in WaveVision 5, the application of these windows, and an elementary discussion on the mathematics behind the calculation of performance metrics such as the SNR.

Introduction

Part 1 of this article included a discussion on finite-time sinusoidal segments and described how data captured from analog-to-digital converters (ADC) cannot be immediately transformed into the frequency domain due to the resulting spectral leakage of the signal. The previous installment also presented frequency coherency as a preferred solution for avoiding spectral leakage but explained how it is impractical in many test scenarios. Applying windowing functions was introduced as the more comprehensive solution to the spectral leakage problem and a few common windowing functions were presented along with their important features. Part 1 also showed that no single windowing function can be used for all applications and that the most common windows are not sufficient for analyzing high resolution ADCs for arbitrary input frequencies due to insufficient window dynamic range.

Part 2 moves the context to windowing functions used in National Semiconductor’s new ADC evaluation software platform called WaveVision 5. Using WaveVision 5, a user can take advantage of a flexible family of Cosine-Sum windowing functions to analyze the performance of ADCs that have a wide span of speeds and resolutions. These windows can be applied to data captured from a variety of different applications. The mathematical foundation of performance calculations in WaveVision 5 is also presented.

The reader will notice that this installment is more mathematically intensive than the previous picture-friendly installment. The intention here is to provide intuitive understanding as well as a more concrete foundation of spectral analysis and provide the mathematical background that can be used in practice.

Family of Cosine-Sum Windows

WaveVision 5 software allows the user to test a wide variety of ADCs with different resolutions from 8-bits to above 16-bits. Due to the wide range of windowing
requirements, WaveVision 5 offers a family of optimized Cosine-Sum windows, see [1], to exercise the tradeoff between main lobe bandwidth (frequency resolution) and side-lobe suppression (dynamic range).

The window family is expressed by (1) where the windows differ by the number of cosine terms and the $A_m$ coefficients used in the summation. Each $A_m$ coefficient is calculated so that the maximum values of all the window side lobes are nearly flat across the spectrum. Actually computing the coefficients is beyond the scope of this article, but a detailed explanation can be found in [1].

The computational complexity of creating a high order, M-term window of this family may seem arduous as a summation of cosinusoids in time, but an N-point DFT of an N-point window reveals that this family of windows can be succinctly described in the sampled frequency domain. Given by (2), the DFT spectrum contains a limited number of non-zero terms whose magnitudes correspond to the $A_m$ coefficients, making the application of the window straight-forward as convolution in the frequency domain.

\[
w(n) = \sum_{m=0}^{M} A_m \cos\left(\pi \frac{n}{N} m \right) \quad \text{for } 0 \leq n \leq \sqrt{-1} \tag{1}
\]

\[
W(f) = \sum_{m=0}^{M} A_m \left[ V \cdot \delta \left[ f - i \cdot \delta \right] \right] \quad \text{for } 0 \leq f \leq \sqrt{-1} \tag{2}
\]

By appending 15N zeros to an N-point window and performing a 16N-point DFT of the window, one can observe a more continuous approximation of the window spectrum to reveal its side lobe suppression. Figure 1 compares the spectrum of the $M = 2$-term, 4-term, 6-term, 8-term, and 11-term windows provided by the WaveVision 5 software and Table 1 compares the main lobe 3-dB bandwidth versus the side lobe suppression.

![Figure 1: Frequency spectrum of cosine-sum windows](image-url)
Table 1: Main lobe bandwidth and Maximum Side Lobe Suppression

<table>
<thead>
<tr>
<th>Cosine-Sum Window Order</th>
<th>Main Lobe 3-dB BW [bins]</th>
<th>Maximum Side Lobe Suppression [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-term</td>
<td>1.31</td>
<td>43.2</td>
</tr>
<tr>
<td>4-term</td>
<td>1.87</td>
<td>98.2</td>
</tr>
<tr>
<td>6-term</td>
<td>2.30</td>
<td>153.6</td>
</tr>
<tr>
<td>8-term</td>
<td>2.65</td>
<td>207.5</td>
</tr>
<tr>
<td>11-term</td>
<td>3.11</td>
<td>289.6</td>
</tr>
</tbody>
</table>

Optimized for maximum side lobe suppression, the 2-term and 4-term windows presented here are also known as the Hamming and Blackman-Nuttall [3] windows respectively. The optimization given by Albrecht in [1] is generalized for any number of cosine terms and contains the Hamming and Blackman-Nuttall windows as members of the family.

Choosing a Window for a Particular Application

Supplying this family of windows in WaveVision 5 allows the user to make a well-informed choice of which window to use depending on the application. All the mystery surrounding the windows and their most important effects is succinctly summarized in the above figure and comparison table.

As an example, National Semiconductor’s ADC14155 has an expected SNR of 71.3 dB relative to full scale (dBFS) and samples at 155 M-samples per second (MSPS). What is the best window for analyzing data from this ADC? With a ~32k sample record the corresponding noise power spectral density is 113.3 dBFS/bin, therefore the 6-term window is sufficient for this application because the side lobe suppression is 153 dB. This choice limits the frequency resolution to 155MHz / 32k-samples * 2 * (window order) = ~57 kHz. The window order is important here because it is tied directly to the main lobe width such that the main lobe of the 11-term window meets the side-lobe level 11 bins away from the lobe center as seen in Figure 1.

An 8-bit, 1 GSPS ADC081000 does not have such a high dynamic range requirement. With an expected SNR of 48.5 dBFS, the noise power spectral density is 90.6 dBFS/bin for a ~32k sample data record. The 98.2 dB side-lobe suppression of the 4-term window is sufficient for this application, limiting the frequency resolution to 244 kHz.

The consequence of using a window with insufficient dynamic range for an application is exemplified in the comparison of Figure 2. Data captured from a high speed, 16-bit converter is windowed with the Flattop window and the 6-term Cosine-Sum window. The comparison clearly show the influence of the side-lobes when the Flattop window is applied, demonstrating the window’s inadequate dynamic range. Note that the spectrum does not have distinct side lobes as in Figure 1. This is because the leakage around the fundamental lobe is made up of one sampled point per side lobe in the sampled spectrum. The result of using the Flattop window in this case for spectral analysis will be a degraded signal-to-noise ratio (SNR) as the side-lobe power will be interpreted as additional noise in the spectrum.
Plotting and Analyzing the Windowed Data Spectrum

After data from a single tone ADC test is appropriately windowed to give sufficient side lobe suppression and after the discrete Fourier transform (DFT) is performed, the spectrum can be plotted and calculations of performance metrics can be made. Here we present two meaningful spectrum normalization schemes and explain the calculation of important values such as the fundamental tone power and total noise power. One scheme is used for easy visualization of the spectrum and the other is used for performance calculations.

When plotting the spectrum, the magnitude is typically normalized in decibels so that 0 dB corresponds to the maximum possible sinusoidal output from the converter. This normalized spectrum for a B-bit converter whose output values range from $-2^{(B-1)}$ to $2^{(B-1)}-1$ is achieved with (4) for an $N$-sample data record $x[n]$ and windowing function $w[n]$. In this case the normalization factor is the window’s coherent gain factor $A_w$ defined in (3). Normalization by the window mean forces the peak value of the fundamental lobe to have the same value for all windows, maintaining visual consistency.¹ This spectrum normalization scheme is used in WaveVision 5 for plotting because a consistent fundamental peak is visually more pleasing and less confusing to most users. This scheme is not used to make performance calculations.

$$A_w = \frac{1}{N} \cdot \sum_{n=0}^{N-1} w[n] = V$$  \hspace{1cm} (3)

¹ This ignores scalloping loss where the peak of the lobe varies depending on the location of the signal frequency between DFT bins. Variation due to scalloping loss reduces for higher order Sum-Cosine windows and is less than 0.9dB for the 4-term window.
Windowing a data record can change the total power in the spectrum due to attenuation at the endpoints, so a different normalization scheme is used for metric calculations to conserve the total signal power when the window is applied. This normalization factor, $P_w$, is the incoherent gain that was briefly mentioned in the previous installment of this article. The incoherent gain is given by (5) and specifies how the total power of a signal changes when the windowing function is applied to the data record where $x[n]$ is the data record, $w[n]$ is the window function, $W[k]$ is the DFT of $w[n]$, and $N$ is the number of points in the data record and window. This normalization ensures that the total power is conserved when the window is applied and is equivalent to scaling the window so that its incoherent gain is unity. The normalized spectrum is given by (6) and is used in WaveVision 5 to make all performance calculations. An additional $\sqrt{2}$ factor is included in the denominator to consider the full dual-sided spectrum, not just the single-sided spectrum.

Performance calculations that are reported as relative to full scale are often of interest. These values, given in units of dBFS, are easily calculated with the normalized spectrum of (6) because only the summing of power is required without needing to know the total power of the fundamental tone. In this case, additional normalization and the calculation of ratios are not required for metrics like the SNR, THD, and SINAD.

\[
\left| X'_w \right| = \log_{10} \left( \frac{2}{N} \cdot \frac{1}{2^{B^-}} \cdot \frac{\text{DFT}_n}{A_w} \right) \cdot w[n] 
\]  

(4)

Now that the spectrum is properly normalized in (6) to make performance calculations, the spectrum must be divided into frequency regions that represent the fundamental, harmonic distortion, or noise. The spectrum contains a large lobe at the fundamental location and smaller lobes at the harmonics, so one must ask the question: How much bandwidth is sufficient to contain the power of a main lobe? Once this is known the SNR, SINAD and THD are easy to calculate.

A helpful characteristic of the presented cosine-sum family is that the main lobe of a window’s frequency spectrum extends $M$ bins from the lobe maximum before reaching the maximum side lobe level where $M$ is the order of the window. Therefore, the approximation can be made that the power of a main lobe spans $2M+1$ bins after applying the window.

Using this approximation, the WaveVision 5 software identifies the bin with the largest power as the fundamental frequency and sums the power in a $2K+1$ bin bandwidth to find the total power of the fundamental. Harmonics of the fundamental are treated similarly. Bins that are not part of a fundamental or harmonic are treated as noise. To
keep the analysis flexible, the user is allowed set \( K \) to any value in the FFT Control Options of the WaveVision 5 software, but it is recommended to set \( K = M \).

The powers of the fundamental tone and total noise are calculated with the dual-sided spectrum in (7) and (8) respectively where \( F_{\text{bin}} \) is the bin containing the peak value of the fundamental lobe, \( 2M+1 \) is the fundamental lobe width, \( N \) is the number of points in the spectrum, and \( X_w''[k] \) is the signal spectrum given by (6). The summation of (8) includes all noise bins in the dual-sided spectrum or equivalently from DC up to the sampling frequency. With the incoherent gain normalization scheme described above, the SNR in units of dBFS is the negative value of \( P_{\text{noise}} \).

\[
P_{\text{fund}} = 0 \cdot \log_{10} \left[ \sum_{k=F_{\text{bin}}+M}^{F_{\text{bin}}+M+1} \left| X_w'' \right|^2 + \sum_{k=N-F_{\text{bin}}+M}^{N-F_{\text{bin}}+M+1} \left| X_w'' \right|^2 \right] \quad \text{[dBFS]} \tag{7}
\]

\[
P_{\text{noise}} = 0 \cdot \log_{10} \left( \sum_{k=\text{NoiseBins}} \left| X_w'' \right|^2 \right) \quad \text{[dBFS]} \tag{8}
\]

The ratio of \( A_w \) and \( P_w \), given by (9), is an important feature of the window itself called the processing gain, \( PG_w \) [2]. Figure 3 shows the zoomed spectra of a coherent frequency signal having no window (Rectangular window), the 4-term, and the 11-term Cosine-Sum window. The change in the peak amplitude after the window is applied is identically the processing gain in [dB]. A Rectangular window gain has a processing gain of unity whereas shaped windows have a processing gain less than one.

One consequence of windowing that can lead to confusion is that the dynamic range in decibel units between the peak of the fundamental lobe and the average noise level will be different depending on the chosen window. In the case of \( P_w \) normalization scheme shown in Figure 3, the average noise level stays the same while the peak value of the fundamental lobe changes. Alternatively, the \( A_w \) normalization scheme locks down the peak value of the fundamental lobe and causes the noise power density to move around for different windows depending on the window’s processing gain, \( PG_w \). The choice to display spectra in WaveVision 5 with the \( A_w \) scheme causes the observed average noise level to be higher than the calculated level that is reported when windowing functions are used. The higher order windows have smaller processing gains and therefore have a larger difference between the observed and calculated average noise level.

With a single data capture this difference is not observable due to the large variation of noise power in adjacent bins but the effect is more apparent when applying FFT averaging, a powerful feature of the WaveVision 5 software. Despite the visual inconsistency in the noise floor, the calculated performance metrics are not affected.

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\footnote{Although scalloping loss affects the main lobe peak value, it affects the total power within the main lobe by less than 0.01 dB for the 4-term and higher order windows making this expression suitable for finding the fundamental power.}
Using the Cosine-Sum Windows

Table 2 gives the standard coefficients for the 4-term and 6-term windows which are most appropriate for today’s state of the art 10-bit to 16-bit ADCs. Using the coefficients and (1), the windows may be calculated and applied to data records that have been imported into data analysis software like Matlab.

Table 2: Sum-Cosine Window Coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>4-term</th>
<th>6-term</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>3.635819267707608e-01</td>
<td>2.935578950102797e-01</td>
</tr>
<tr>
<td>a₁</td>
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<td>4.519357723474506e-01</td>
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<td>a₂</td>
<td>1.365995139786921e-01</td>
<td>2.014164714263962e-01</td>
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<tr>
<td>a₃</td>
<td>1.064112210553003e-02</td>
<td>4.792610922105837e-02</td>
</tr>
<tr>
<td>a₄</td>
<td>5.026196426859393e-03</td>
<td>1.375556795588777e-04</td>
</tr>
</tbody>
</table>

Conclusion

ADC software evaluation platforms supplied to customers are designed to be as user friendly as possible, but pitfalls are always present. Providing easy-to-use options
will greatly reduce the potential for frustration, but education must also be provided to avoid erroneous evaluation results. When combined with this article, National Semiconductor’s new WaveVision 5 software accomplishes both. The family of Cosine-Sum windows provided in the software makes selection of the appropriate window for an application as simple as looking at a chart due to each window’s basic form and intuitive results. A firm, mathematical background has also been provided here to increase a user’s understanding of the analyses built into WaveVision 5 as well as to allow the skeptics to crunch the numbers themselves.

References


About the Author

Josh Carnes is an applications engineer with National Semiconductor’s Strategic Signal Path Group, based in Ft. Collins, Colorado. He received his BSEE and MSEE degrees from Oregon State University in 2004 and 2007, respectively, with research focusing on low-voltage pipelined ADC design techniques. His interests include cellular base station subsystems, wireless communications, as well as automated testing and analysis of ADCs.