Microwave and RF Design: A Systems Approach
4.1 Introduction

A transmission line stores electric and magnetic energy distributed in space and alternating between the two forms in time. That is, at any position along the line the energy is stored in a combination of electric and magnetic forms and, for an alternating signal at any position on the line, converted from one form to the other as time progresses. As such, a transmission line has a circuit form that combines inductors, $L_s$ (for the magnetic energy), capacitors, $C_s$ (for the electric energy), and resistors, $R_s$ (modeling losses), whose values are dependent on the geometry of the line and the properties of the materials comprising the line. Thus transmission lines of various lengths and crosssections mimic circuits. Distributed structures,
of which transmission lines are the most fundamental members, are what distinguishes RF, microwave, packaging, and high-speed digital design from lumped-element ($R$, $L$, and $C$) circuit design. In this chapter the properties of transmission lines are considered. It will be seen how they can be modeled using lumped elements and it will be seen how simple lumped-element circuits can be realized using combinations of transmission lines, and how transmission lines can be used to achieve surprising functionality beyond that which can be achieved with lumped-element circuits. The discussion of transmission lines introduces concepts that apply to all distributed structures.

The transmission lines considered here are systems of two or more closely spaced parallel conductors. For now, the discussion is restricted to considering just two parallel conductors, as shown in Figure 4-1(a), with the distance between the two wires being substantially smaller than the wavelengths of the signals on the line. Then the structure may be satisfactorily analyzed on the basis of voltages and currents. As the frequency increases, and therefore the wavelength becomes smaller, and the

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1 Rectangular waveguide is a transmission line that has just one conductor. Dielectric lines can have no conductor.
crosssectional dimensions become electrically larger, it becomes necessary
to set up a complete EM field solution in order to analyze the structures. By
considering only transmission lines where electrically small transverse line
dimensions\(^2\) prevail, a number of useful results are obtained on a voltage
and current basis. The theory thus developed is called distributed circuit
theory.

**Metallic conductors** forming a transmission line are **crystals** with
positively charged **ions** locked into position in a regular lattice. Each
ion comprises a nucleus of an atom which is positively charged and a
complement of electrons local to each atom which almost, but not quite,
balances the positive charge locally. In a metallic crystal there are free
**electrons** shared by several ions, with the overall effect that the positive and
negative charges are balanced and the free electrons can wander around
the lattice. The movement of charge results in a **magnetic field**, and hence
**magnetic energy** storage. The ability of a structure to store magnetic energy
is described by its inductance, \(L\). Similarly the rearrangement of charge
to produce localized net positive or negative charge on one conductor of
the pair is matched by an opposing charge on the adjacent conductor. This
separation of charge results in an **electric field**, and thus **electric energy**
storage, with the capacitance, \(C\), indicating the amount of energy that can
be stored. The ratio of the energy stored in the magnetic and electric forms
is proportional to \(L/C\) and the rate at which the energy can be moved is
proportional to \(1/\sqrt{LC}\); together these determine the characteristics of the
transmission line. A lossless transmission line is generally characterized by
its **characteristic impedance**:

\[
Z_0 = \sqrt{\frac{L}{C}},
\tag{4.1}
\]

with the units of ohms (\(\Omega\)) and its propagation constant

\[
\gamma = \sqrt{LC},
\tag{4.2}
\]

which in the SI system has the units of inverse meters (m\(^{-1}\)). Another
way of looking at transmission lines is that they confine and guide an EM
field between them. In fact, transmission lines are also, but less commonly,
called waveguides. Many structures can successfully guide waves, and two
additional ones are shown in Figures 4-1(b) and 4-1(c). The “strip-above-
ground” transmission line shown in Figure 4-1(b) confines the EM field
mostly between the flat metal strip and the metallic ground plane below
it. However, some energy is distributed above and to the sides of the lines
so that the EM energy is not completely confined. As long as the distance
between the strip and the ground plane is less than half a wavelength of
the signal on the line, the energy will follow the strip along its length and

\(^2\) Say, less than 1/20 of a wavelength.
around bends, etc. When the separation is more than one-half wavelength, EM energy will radiate away from the line. The coaxial line shown in Figure 4-1(c) completely confines the field between the inner conductor and the outer conductor. Now, if the spacing between the conductors is less than one-half wavelength the fields will, in general, be coherently guided in what is called a single mode. With some structures, such as the strip-above-ground line of Figure 4-1(b), there is an additional criterion that the width of the strip be less than $\lambda/2$.

In low-frequency analog and digital circuits, transmission lines are often referred to as **interconnects** and can be viewed simply as wires, and provided that the wire has sufficiently low resistance, the interconnect can be largely ignored. However, if transmission must be over a nonnegligible distance compared to a **wavelength** ($\lambda$), then the interconnect must be considered as part of the circuit.

The earliest fundamental understanding of signal transmission led to telegraphy over distances. The critical theoretical step that enabled transmission over more than short distances to be achieved was the development of an understanding of signal transmission on lines using what is now called **phasor analysis**. The mathematician and engineer Oliver Heaviside [50] developed the frequency-domain-based treatment of signal propagation on transmission lines. Frequency-domain analysis (i.e., using phasors) is still the best way to develop a fundamental understanding of transmission lines, even if they are used for purely digital signals.

The key determinant of whether a transmission line can be considered as an invisible connection is whether the signal anywhere along the interconnect has the same value at a particular instant. If the value of the signal (say, voltage) varies along the line (at an instant), then it may be necessary to consider transmission line effects. A typical criterion used is that if the length of the interconnect is less than 1/20 of the wavelength of the highest-frequency component of a signal, then transmission line effects can be safely ignored and the circuit can be modeled as a single **RLC** circuit [51]. The actual threshold used—$\lambda/20$, $\lambda/10$, or $\lambda/5$—is based on experience and the particular application. For example, an interconnect on a silicon chip clocking at 4 GHz has an appreciable frequency component at 20 GHz. Then the interconnect reaches the $\lambda/10$ threshold when it is 4.5 mm long. This is less than the dimensions of most chips, which can be up to 2 cm on a side. Thus it takes a finite time for the variation of a voltage at one end of an interconnect to impact the voltage at the other end. The ultimate limit

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3 A phasor is a complex number that combines the amplitude and phase of a sinewave. With transmission line equations, the introduction of phasors eliminates time dependence from the equations and the dimensionality of the equations is reduced by one. Also, in circuit analysis, when phasors are introduced, differential equations in time become algebraic equations, which are much easier to handle. When phasors are used, no information on frequency is retained.
is determined by the speed of light, $c$, but this is reduced by the relative permittivity and permeability of the material in which the fields exist. The relative permittivity and permeability describe the effect of excess potential energy storage in the material in a manner that is analogous to storing mechanical energy in a spring.

### 4.1.1 Movement of a Signal on a Transmission Line

A **coaxial line** (Figure 4-2(a)) is the quintessential transmission line, as it is one of the few transmission line structures that can be described exactly from first principles when there is no loss. Here a realistic coaxial line is considered with conductors having a small amount of loss, a structure that does not have an exact solution. When a positive voltage pulse is applied to the center conductor of the coaxial line, an electric field results that is essentially directed from the center conductor to the outer conductor. A much smaller component of the electric field will also be directed along the line. The direction of the electric field is the direction in which a positive charge would move if it was released into the field. The component of the field that is directed along the shortest path from the center conductor to the outer conductor (in what is called the **transverse plane** in Figure 4-2(b) shows the fields in the structure after the pulse has started moving along the line. This is shown in another view in Figure 4-3. The transverse voltage, $V_T$, is given by $E_T(\alpha - \beta)$. This is a good measure, provided that the transverse dimensions are sufficiently small compared to a wavelength (otherwise the integral does not come out so simply, as it is path dependent).

The pulse moves down the lossless line at the **phase velocity**, $v_p$.\(^5\) It is determined by the physical properties of the region between the conductors. The permittivity, $\varepsilon$, describes the energy storage capability associated with the electric field, $E$, and the energy storage associated with magnetic field, $H$, is described by the permeability, $\mu$. (Both $\varepsilon$ and $\mu$ are properties of the

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\(^4\) Applying Maxwell's equations and assuming perfect conductors and using cylindrical symmetry and phasors reduces the dimensionality of the problem from four dimensions (the three spatial dimensions plus time) to two spatial dimensions. Maxwell's equations are discussed in Appendix D.

\(^5\) Specifically, the phase velocity is the apparent velocity of a point of constant phase on a sinewave. While $v_p$ can vary with frequency, it is almost frequency independent for a low-loss coaxial line of relatively small transverse dimensions (less than $\lambda/10$).
Figure 4-2  A coaxial transmission line: (a) three-dimensional view; (b) the line with pulsed voltage source showing the electric fields at an instant in time as a voltage pulse travels down the line.

\[ v_p = \frac{1}{\sqrt{\mu \varepsilon}} \]  

(4.3)

Figure 4-3  Fields, currents, and charges on the coaxial transmission line of Figure 4-2.

\( Q_{\text{CENTER}} \) is the net free charge on the center conductor.
\( I_{\text{CENTER}} \) is the current on the center conductor.
\( t_4 > t_3 > t_2 > t_1 \)

medium—the material.) It has been determined that

In a vacuum \( \varepsilon = \varepsilon_0 \), the free-space permittivity, and \( \mu = \mu_0 \), the free-space

\( 6 \) This is derived from Maxwell's equations; there is no underlying theory for these equations, but they have been verified by numerous experiments.
permeability. These are physical constants and have the values:

\[
\begin{align*}
\text{Permittivity of free space} & \quad \varepsilon_0 = 8.854 \times 10^{-12} \text{ F} \cdot \text{m}^{-1} \\
\text{Permeability of free space} & \quad \mu_0 = 4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1}
\end{align*}
\]

One conclusion here is that EM energy can be stored in a vacuum (or free space). So in free space, or on an air-filled coaxial line, \(v_p = c = 1/\sqrt{\mu_0 \varepsilon_0} = 3 \times 10^8 \text{ m} \cdot \text{s}^{-1}\). The wavelengths, \(\lambda_0 = c/f\), at several different frequencies are

<table>
<thead>
<tr>
<th>(f)</th>
<th>100 MHz</th>
<th>1 GHz</th>
<th>10 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_0)</td>
<td>3 m</td>
<td>30 cm</td>
<td>3.0 cm</td>
</tr>
</tbody>
</table>

These are dimensions comparable to the sizes of many circuits. Commonly \(\lambda_0\) is used to indicate the wavelength in free space and \(\lambda_g\), the guide wavelength, is used to denote the wavelength of a transmission line (or waveguide). A dielectric cannot have a dielectric constant less than \(\varepsilon_0\), and so it is convenient to use the relative permittivity (or the less commonly used term dielectric constant), \(\varepsilon_r\), defined as

\[
\varepsilon_r = \varepsilon/\varepsilon_0.
\]

Similarly the relative permeability is

\[
\mu_r = \mu/\mu_0,
\]

and most materials have \(\mu_r = 1\). The permittivity, permeability, and conductivity of materials used in RF and microwave circuits are given in Appendix C.

### Example 4.1 Transmission Line Wavelength

A length of coaxial line is filled with a dielectric having a relative dielectric constant of 20 and is designed to be one-quarter wavelength long at frequency, \(f\), of 1.850 GHz.

(a) What is the free-space wavelength at 1850 MHz?

(b) What is the wavelength of the signal in the dielectric-filled coaxial line?

(c) How long is the line?

**Solution:**

(a) \(\lambda_0 = c/f = 3 \times 10^8/1.85 \times 10^9 = 0.162 \text{ m} = 16.2 \text{ cm}\)

(b) Note that for a dielectric filled line with \(\mu_r = 1\), thus \(\lambda = v_p/f = c/(\sqrt{\varepsilon_r}) = \lambda_0/\sqrt{\varepsilon_r}\), so \(\lambda = \lambda_0/\sqrt{\varepsilon_r} = 16.2 \text{ cm}/\sqrt{20} = 3.62 \text{ cm}\)

(c) \(\lambda_g/4 = 3.62 \text{ cm}/4 = 9.05 \text{ mm}\)
4.1.2 Current and Voltage on Transmission Lines

The majority of transmission lines used in design are planar, as these can be defined using masks, photoresist, and etching of metal sheets. Such lines are called planar interconnect. A common planar interconnect is the microstrip line shown in crosssection in Figure 4-4. This crosssection is typical of what would be found on a semiconductor or printed wiring board (PWB), which is also called a printed circuit board (PCB). Current flows in both the top and bottom conductor, but in opposite directions. The physics is such that if there is a signal current on the top conductor, there must be a return signal current, which will tend to be as close to the signal current as possible to minimize stored energy. The provision of a signal return path close to the signal path is important in maintaining the integrity (i.e., predictable signal waveform) of an interconnect.

In the microstrip line, electric field lines start on one of the conductors and finish on the other and are located almost entirely in the plane transverse to the long length of the line. The magnetic field is also mostly confined to the transverse plane, and so this line is referred to as a transverse electromagnetic (TEM) line. Integrating the electric field along a path gives the voltage. Since the voltage between the top and bottom conductors is more or less the same everywhere, longer $E$ field lines correspond to lower levels of $E$ field. The strength of the $E$ field is also indicated by the density of the $E$ field lines. This is a drawing convention for electric and magnetic fields. A further comment is warranted for this line. This line is more commonly called a quasi-TEM line, as the longitudinal fields are not as negligible as with the coaxial line considered previously. On a microstrip line, the relative level of the longitudinal fields increases with frequency, but below about 10 GHz and for typical dimensions used, the line is still essentially TEM. Figure 4-4 illustrates an important point: current flows in the strip and a return current flows in what is normally regarded as the grounded conductor. Both the signal and return currents induce a magnetic field and the closed path integral of the magnetic field is equal to the current contained within the path.

Various schematic representations of a transmission line are used. To illustrate this, consider the various representations in Figure 4-5 of a length of microstrip line shorted by a via at the end denoted by “2” (specifically the “2” refers to Port 2). The representations shown in Figures 4-5(d), 4-5(e), and 4-5(f) are commonly used in circuit diagrams. The preference depends on the number of transmission lines in a circuit diagram. If a circuit diagram has many transmission line elements, such as the filter circuits of Chapter 10, then the simple representation of Figure 4-5(f) is most common. If there are

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7 An entertaining account of the early application of microstrip transmission lines can be found in Barrett [52]. The original analysis of microstrip was based on the unfolding of a coaxial line.
Figure 4-4 Crosssectional view of a microstrip line showing the electric and magnetic field lines and current flow for a microstrip interconnect. The electric and magnetic fields are in two media—the dielectric and air.

Figure 4-5 Representations of a shorted microstrip line with a short (or via) at Port 2: (a) three-dimensional (3D) view indicating via; (b) side view; (c) top view with via indicated by X; (d) schematic representation of transmission line; (e) alternative schematic representation; and (f) representation as a circuit element.

few transmission line elements, then the representation of Figure 4-5(d) is most common.

4.1.3 Forward- and Backward-Traveling Pulses

Forward- and backward-traveling pulses are shown in Figure 4-6(a), for the situation where the resistance at the end of the line is lower than the characteristic impedance of the line (\( Z_L < Z_0 \)). The voltage source is a step voltage which is zero for time \( t < 0 \). At time \( t = 0 \), the step is applied to the line and it begins traveling down the line, as shown at time \( t = 1 \). This voltage step moving from left to right is called the forward-traveling voltage wave. At time \( t = 2 \), the leading edge of the step reaches the load, and as the load has lower resistance than the characteristic impedance of the line, the total voltage across the load drops below the level of the forward-traveling voltage step. The reflected wave is called the backward-traveling wave and it must be negative, as it adds to the forward-traveling wave to yield the total voltage. Thus the voltage reflection coefficient, \( \Gamma \), is negative and the total voltage on the line, which is all we can directly observe, drops. A reflected, smaller, and opposite step signal travels in the backward direction and adds to the forward-traveling step to produce the waveform shown at \( t = 3 \). The
impedance of the source is matched to the transmission line impedance so that the reflection at the source is zero. The signal on the line at time $t = 4$, the round-trip propagation time of the line, therefore remains at the lower value. The easiest way to remember the polarity of the reflected pulse is to consider the situation with a short-circuit at the load. Then the total voltage on the line at the load end must be zero. The only way this can occur when a signal is incident is if the reflected signal is equal in magnitude but opposite in sign, in this case $\Gamma = -1$. So whenever $|Z_L| < |Z_0|$, the reflected pulse will tend to subtract from the incident pulse. You will note that in Figure 4-6(a) a schematic symbol is used for the line. Even though it appears that just one conductor is shown, this symbol represents the full transmission line with the signal path and the signal return path.

The opposite situation occurs when the resistance at the load end is higher
than the characteristic impedance of the line (Figure 4-6(b)). In this case the reflected pulse has the same polarity as the incident signal. Again, to remember this, think of the open-circuited case. The voltage across the load does not need to be zero, and indeed doubles, as the reflected pulse has the same sign as well as magnitude as that of the incident signal, in this case $\Gamma = +1$.

A more illustrative situation is shown in Figure 4-7, where a more complicated signal is incident on a load that has a resistance higher than that of the characteristic impedance of the line. The peaking of the voltage that results at the load is typically the design objective in many long digital interconnects, as less overall signal energy needs to be transmitted down the line, or equivalently a lower current drive capability of the source is required to achieve first incidence switching. This is at the price of having reflected signals on the interconnects, but these can be dissipated through a combination of the interconnect loss and absorption of the reflected signal at the driver.

### 4.2 Media

Design involves choosing the transmission line structure to use and the substrate. In this section, the electrical properties of materials will be discussed and then substrates commonly used with planar interconnects will be considered.

#### 4.2.1 Dielectric Effect

The presence of material between the conductors alters the electrical characteristics of the interconnect. With a dielectric, the application of an electric field moves the centers of positive and negative charge at the atomic and molecular level. Moving the charge centers changes the amount of energy stored in the electric field—a process akin to storing energy in a stretched spring. The extra energy storage property is described by the relative permittivity, $\varepsilon_r$, which is the ratio of the permittivity of the material to that of free space:

$$\varepsilon = \varepsilon_r \varepsilon_0 .$$

The relative permittivities of materials commonly used with interconnects range from 2.08 for Teflon™, used in high-performance PCBs and coaxial cables, to 11.9 for silicon (Si), to 3.8–4.2 for silicon dioxide (SiO$_2$), and 12.4 for gallium arsenide (GaAs). Values of permittivity for other materials are given in Table 4-1.

When the fields are in more than one medium (a nonhomogeneous transmission line), as for the microstrip line shown in Figure 4-4, the effective relative permittivity, $\varepsilon_{\text{eff}}$, is used. The characteristics of the nonhomogeneous line are then more or less the same as for the same