Figure 4-32  Rat-race hybrid with input at Port 1, outputs at Ports 3 and 4, and virtual ground at Port 2: (a) implementation as a planar circuit; (b) transmission-line model; and (c) equivalent circuit model.

4.13 Resonators

In a lumped-element resonant circuit, stored energy is transferred between an inductor which stores magnetic energy and a capacitor which stores electric energy, and back again every period. Microwave resonators function the same way, exchanging energy stored in electric and magnetic forms, but with the energy stored spatially. Resonators are described in terms of their quality factor,

\[ Q = 2\pi f_0 \left( \frac{\text{average energy stored in the resonator at } f_0}{\text{power lost in the resonator}} \right), \]  

(4.197)

where \( f_0 \) is the resonant frequency. The \( Q \) is reduced and thus the resonator bandwidth is increased by the power lost to the external circuit so that the
TRANSMISSION LINES

Figure 4-33  Microwave resonators: (a) resonator response, (b) rectangular cavity resonator, (c) microstrip patch resonator (d) microstrip gap-coupled reflection resonator, (e) transmission dielectric transmission resonator in microstrip, (f) parallel equivalent circuits, and (g) series equivalent circuits.

loaded $Q$ is

$$ Q_L = \frac{2\pi f_0}{1} \left( \frac{\text{average energy stored in the resonator at } f_0}{\text{power lost in the resonator and to the external circuit}} \right) = \frac{1}{1/Q + 1/Q_X}, \quad (4.198) $$

where $Q_X$ is called the external $Q$. $Q_L$ accounts for the power extracted from the resonant circuit. For the simple response shown in Figure 4-33(a), the half power (3 dB) bandwidth is $f_0/Q_L$. Near resonance the response of a microwave resonator is very similar to the resonance response of a parallel or series $LC$ resonant circuit, shown in Figures 4-33(f) and 4-33(g). These equivalent circuits can be used over a narrow frequency range.

Several types of resonators are shown in Figure 4-33(b). Figure 4-33(b) is a rectangular cavity resonator coupled to an external coaxial line by a small coupling loop. Figure 4-33(c) is a microstrip patch reflection resonator. This resonator has large coupling to the external circuit. The coupling can be reduced and photolithographically controlled by introducing a gap, as shown in Figure 4-33(d), for a microstrip gap-coupled transmission line reflection resonator. The $Q$ of a resonator can be dramatically increased by using a low-loss, high dielectric constant material, as shown in Figure 4-33(e), for a dielectric transmission resonator in microstrip. Here the resonant frequency of a rectangular cavity is varied by changing the physical dimensions of the cavity, with the null of the detector indicating the cavity resonant frequency.
4.14 Summary

In this chapter a classical treatment of transmission lines was presented. Transmission lines are distributed elements and form the basis of microwave circuits. A distinguishing feature is they support forward- and backward-traveling waves. This chapter provided an understanding of signals on transmission lines. The next chapter points out problems that can occur and provides intuitive understanding and guidelines for the design of transmission lines and interconnects.

4.15 Exercises

1. A coaxial line is short-circuited at one end and is filled with a dielectric with a relative dielectric constant of 64. It is used at a frequency, \( f_0 \), of 18 GHz. [Parallels Example 4.1 on Page 167]
   (a) What is the free-space wavelength at 18 GHz?
   (b) What is the wavelength in the dielectric-filled coaxial line at 18 GHz?
   (c) The first resonance of the coaxial resonator occurs when it is one-quarter wavelength long. How long is the resonator at this first resonance?

2. Design a microstrip line having a 50 \( \Omega \) characteristic impedance. The substrate has a dielectric constant of 2.3 and is 250 \( \mu \)m thick. The operating frequency is 18 GHz. You need to determine the width of the microstrip line.

3. A transmission line has the per unit length parameters: \( R = 2 \Omega/\text{cm}, \; L = 100 \text{nH}\cdot\text{m}^{-1}, \; G = 1 \text{mS}\cdot\text{m}^{-1}, \; C = 200 \text{pF}\cdot\text{m}^{-1}. \)
   (a) What is the propagation constant of the line at 5 GHz?
   (b) What is the characteristic impedance of the line at 5 GHz?
   (c) Plot the magnitude of the characteristic impedance versus frequency from 100 MHz to 10 GHz.

4. A transmission line has an attenuation of 2 dB\cdot m\(^{-1}\) and a phase constant of 25 radians\cdot m\(^{-1}\) at 2 GHz. [Parallels Example 4.5 on Page 187]
   (a) What is the complex propagation constant of the transmission line?
   (b) If the capacitance of the line is 50 pF\cdot m\(^{-1}\) and the conductive loss is zero (i.e., \( G = 0 \)), what is the characteristic impedance of the line?

5. A transmission line has the following \( RLCG \) parameters: \( R = 100 \Omega\cdot \text{m}^{-1}, \; L = 85 \text{nH}\cdot\text{m}^{-1}, \; G = 1 \text{S}\cdot\text{m}^{-1}, \; \text{and} \; C = 150 \text{pF}\cdot\text{m}^{-1}. \) Consider a traveling wave on the transmission line with a frequency of 1 GHz. [Parallels Example 4.4 on Page 184]
   (a) What is the attenuation constant?
   (b) What is the phase constant?
   (c) What is the phase velocity?
   (d) What is the characteristic impedance of the line?
   (e) What is the group velocity?
   (f) If the line resistance is \( R = 0 \Omega\cdot \text{m}^{-1}, \) what is the phase velocity?
   (g) If the line resistance is \( R = 0 \Omega\cdot \text{m}^{-1}, \) what is the group velocity?
   (h) If the line resistance is \( R = 10 \text{k}\Omega\cdot \text{m}^{-1}, \) what is the phase velocity?
   (i) If the line resistance is \( R = 10 \text{k}\Omega\cdot \text{m}^{-1}, \) what is the group velocity?

6. A very low-loss microstrip transmission line has the following per unit length parameters: \( R = 2 \Omega\cdot \text{m}^{-1}, \; L = 80 \text{nH}\cdot\text{m}^{-1}, \; C = 200 \text{pF}\cdot\text{m}^{-1}, \; \text{and} \; G = 1 \mu\text{S}\cdot\text{m}^{-1}. \)
   (a) What is the characteristic impedance of the line if loss is ignored?
   (b) What is the attenuation constant due to conductor loss?
   (c) What is the attenuation constant due to dielectric loss?
7. A lossless transmission line has the following per unit length parameters: $L = 80 \text{ nH m}^{-1}$, $C = 200 \text{ pF m}^{-1}$. Consider a traveling wave on the transmission line with a frequency of 1 GHz.

(a) What is the attenuation constant?
(b) What is the phase constant?
(c) What is the phase velocity?
(d) What is the characteristic impedance of the line?
(e) Now consider that the dielectric is replaced by a dielectric with $\varepsilon_r = 1$ (or air). The capacitance per unit length of the line is now $C(\text{air}) = 50 \text{ pF m}^{-1}$. What is the effective relative dielectric constant of the line?

8. The resonator below is constructed from a 3.0 cm length of 100 $\Omega$ air-filled coaxial line, shorted at one end and terminated with a capacitor at the other end.

(a) What is the lowest resonant frequency of this circuit without the capacitor (ignore the 10 k$\Omega$ resistor)?
(b) What is the capacitor value to achieve the lowest-order resonance at 6.0 GHz (ignore the 10 k$\Omega$ resistor)?
(c) Assume that loss is introduced by placing a 10 k$\Omega$ resistor in parallel with the capacitor. What is the $Q$ of the circuit?
(d) Approximately what is the bandwidth of the circuit?

9. A transmission line has an attenuation of 0.2 dB/cm and a phase constant of 50 radians m$^{-1}$ at 1 GHz.

(a) What is the complex propagation constant of the transmission line?
(b) If the capacitance of the line is 100 pF m$^{-1}$ and the conductive loss is zero (i.e., $G = 0$), what is the complex characteristic impedance of the line?
(c) If the line is driven by a source modeled as an ideal voltage and a series impedance, what is the impedance of the source for maximum transfer of power to the transmission line?
(d) If 1 W is delivered to the transmission line by the generator, what is the power in the forward-traveling wave on the line at 2 m from the generator?

10. A transmission line is driven by a 1 GHz generator having a Thevenin equivalent impedance of 50 $\Omega$. The transmission line is lossless, has a characteristic impedance of 75 $\Omega$, and is infinitely long. The maximum power that can be delivered to a load attached to the generator is 2 W.

(a) What is the total (phasor) voltage at the input to the transmission line?
(b) What is the magnitude of the forward-traveling voltage wave at the generator side of the line?
(c) What is the magnitude of the forward-traveling current wave at the generator side of the line?

11. A transmission line has a characteristic impedance $Z_0$ and is terminated in a load with a reflection coefficient of $0.8 \angle 45^\circ$. A forward-traveling voltage wave on the line has a power of 1 dBm.

(a) How much power is reflected by the load?
(b) What is the power delivered to the load?

12. A 50 $\Omega$ transmission line is terminated in a load that results in a reflection coefficient of $0.5 + j0.5$.

(a) What is the load impedance?
(b) What is the VSWR on the line?
(c) What is the input impedance if the line is one-half wavelength long?
13. A transmission line has a characteristic impedance $Z_0$ and is terminated in a load with a reflection coefficient of 0.8. A forward-traveling voltage wave on the line has a power of 1 W.

(a) How much power is reflected by the load?
(b) What is the power delivered to the load?

14. Communication filters are often constructed using several shorted transmission line resonators that are coupled by passive elements such as capacitors. Consider a coaxial line that is short-circuited at one end. The dielectric constant filling the coaxial line has a relative dielectric constant of 64 and the resonator is to be designed to resonate at a center frequency, $f_0$, of 800 MHz. [Parallels Example 4.15 on Page 209]

(a) What is the wavelength in the dielectric-filled coaxial line?
(b) What is the form of the equivalent circuit (in terms of inductors and capacitors) of the quarter-wavelength long resonator if the coaxial line is lossless?
(c) What is the length of the resonator?
(d) If the diameter of the inner conductor of the coaxial line is 2 mm and the inside diameter of the outer conductor is 5 mm, what is the characteristic impedance of the coaxial line?
(e) Calculate the input admittance of the dielectric-filled coaxial line at 0.99$f_0$, $f_0$, and 1.01$f_0$. Determine the numerical derivative of the line admittance at $f_0$.
(f) Derive the numeric values of the equivalent circuit of the resonator at the resonant frequency and derive the equivalent circuit of the resonator. Hint: Match the derivative expression derived in (e) with the actual derivative derived in Example 4.15.

15. A load consists of a shunt connection of a capacitor of 10 pF and a resistor of 25 Ω. The load terminates a lossless 50 Ω transmission line. The operating frequency is 1 GHz. [Parallels Example 4.7 on Page 196]

(a) What is the impedance of the load?
(b) What is the normalized impedance of the load (normalized to the characteristic impedance of the line)?
(c) What is the reflection coefficient of the load?
(d) What is the current reflection coefficient of the load? (When the term reflection coefficient is used without a qualifier it is assumed to be the voltage reflection coefficient.)
(e) What is the standing wave ratio (SWR)?
(f) What is the current standing wave ratio (ISWR)? (When SWR is used on its own it is assumed to refer to the voltage standing wave ratio [VSWR].)

16. The transmission line shown in the Figure 4-16 consists of a source with Thevenin impedance $Z_1 = 40$ Ω and source $E = 5$ V (peak) connected to a quarter-wavelength long line of characteristic impedance $Z_{01} = 50$ Ω, which in turn is connected to an infinitely long line of characteristic impedance $Z_{02} = 100$ Ω. The transmission lines are lossless. Two reference planes are shown in Figure 4-16. At reference plane 1 the incident power is $P_{I1}$, the reflected power is $P_{R1}$, and the transmitted power is $P_{T1}$. $P_{I2}$, $P_{R2}$, and $P_{T2}$ are similar quantities at reference plane 2. [Parallels Examples 4.9 and 4.10 on Pages 198 and 200]

(a) What is $P_{I1}$?
(b) What is $P_{R2}$?

17. A transmission line is driven by a 1 GHz generator with a Thevenin equivalent impedance of 50 Ω. The maximum power that can be delivered to a load attached to the generator is 2 W. The generator is connected to a 10 cm long lossless transmission line with a characteristic impedance of 75 Ω. Finally, the line is terminated in a load that has a complex reflection coefficient (referred to 50 Ω) of $0.65 + j0.65$. The effective relative permittivity, $\varepsilon_{eff}$, of the transmission line medium is 2.0, and the effective relative permeability of the line is that of free space.
(a) Calculate the forward-traveling voltage wave (at the generator end of the transmission line). Ignore reflections from the load at the end of the 75 Ω line.

(b) What is the load impedance?

(c) What is the wavelength of the forward-traveling voltage wave?

(d) What is the VSWR on the line?

(e) What is the propagation constant of the transmission line?

(f) What is the input reflection coefficient (at the generator end) of the line?

(g) What is the power delivered to the load?

18. A shorted coaxial line is used as a resonator. The first resonance is determined to be a parallel resonance and is at 1.4 GHz, and in a standard resonator test fixture the unloaded Q is determined to be 520.

(a) Draw the lumped-element equivalent circuit of the resonator.

(b) How long is the resonator in terms of wavelength?

(c) Briefly describe how energy is stored in the resonator.

(d) Briefly describe the sources of loss in the resonator.

(e) Now the resonator is used in an actual application and the 3 dB bandwidth is found to be 0.4%. What is the external (sometimes called extrinsic) Q, Qe, of the resonator in this application? (First you will need to determine the loaded Q, i.e., QL.)

19. A transmission line is driven by a generator with a maximum available power of 23 dBm and a Thevenin equivalent impedance of 60 Ω. The transmission line has a characteristic impedance of 25 Ω. [Parallels Example 4.13 on Page 208]

(a) What is the Thevenin equivalent generator voltage?

(b) What is the magnitude of the forward-traveling voltage wave on the line? Assume the line is infinitely long.

(c) What is the power of the forward-traveling voltage wave?

20. A 50 Ω air-filled transmission line is connected between a 40 GHz source with a Thevenin equivalent impedance of 50 Ω and a load. The SWR on the line is 3.5.

(a) What is the magnitude of the reflection coefficient, ΓL, at the load.

(b) What is the phase constant, β, of the line?

(c) If the first minimum of the standing wave voltage on the transmission line is at a distance 2 mm from the load, determine the electrical distance (in degrees) of the SWR minimum from the load angle of the ΓL at the load.

(d) Determine the angle of ΓL at the load.

(e) What is ΓL in magnitude-phase form?

(f) What is ΓL in complex (rectangular) form?

(g) Determine the load impedance, ZL.

21. A load has an impedance ZL = 45 + j75. The reference system impedance is 100 Ω.

(a) What is the reflection coefficient?

(b) What is the current reflection coefficient?

(c) What is the SWR?

(d) What is the ISWR?

(e) The power available from a source with a 100 Ω Thevenin equivalent impedance is 1 mW. The source is connected directly to the load ZL. Calculate the power delivered to ZL using the reflection coefficient?

(f) What is the total power absorbed by the Thevenin equivalent source impedance?

(g) Discuss the effect of inserting a lossless 100 Ω transmission line between the source and the load.

22. A load consists of a resistor of 100 Ω in parallel with a 5 pF capacitor with an electrical signal at 2 GHz.

(a) What is the load impedance?

(b) What is the reflection coefficient in a 50 Ω reference system?

(c) What is the SWR on a 50 Ω transmission line connected to the load?
(d) Develop an analytic formula relating the reflection coefficient ($\Gamma_1$) in one reference system ($Z_{01}$) to the reflection coefficient ($\Gamma_2$) in another reference system ($Z_{02}$).

(e) Develop an analytic formula relating the SWR in one reference system (SWR$_1$) to the SWR (SWR$_2$) in another reference system.

(f) Calculate the SWR on a 100 $\Omega$ line.

23. An amplifier is connected to a load by a transmission line matched to the amplifier. If the SWR on the line is 1.5, what percentage of the available amplifier power is absorbed by the load?

24. The output amplifier of a cellular phone can tolerate a mismatch characterized by a maximum SWR of 2.0. The amplifier is characterized by a Thevenin equivalent circuit with an impedance of 50 $\Omega$ and is connected directly to an antenna characterized by a load impedance, $Z_L$. Determine the tolerance limits on $Z_L$ so that the amplifier does not self-destruct (consider that $Z_L$ is real).

25. A source is connected to a load by a one-wavelength long transmission line having a loss of 1.5 dB. The source reflection coefficient (referred to the transmission line) is 0.2 and the load reflection coefficient is 0.5.

(a) What is the transmission coefficient?

(b) Draw the bounce diagram using the transmission and reflection coefficients. Determine the overall effective transmission coefficient from the source to the load. Calculate the power delivered to the load from a source with an available power of 600 mW.

26. The resonator below is constructed from a 3.0 cm length of 100 $\Omega$ air-filled coaxial line, shorted at one end and terminated with a capacitor at the other end, as shown:

\[ R = 10^4 \Omega \]

\[ Z_0 = 100 \Omega \]

(a) What is the lowest resonant frequency of this circuit without the capacitor (ignore the resistor)?

(b) What is the capacitor value to achieve resonance at 6.0 GHz?

(c) Assume that loss is introduced by placing a 10 k$\Omega$ resistor in parallel with the capacitor. What is the $Q$ of the circuit?

(d) What is the bandwidth of the circuit?

27. A load of 100 $\Omega$ is to be matched to a transmission line with a characteristic impedance of 50 $\Omega$. Use a quarter-wave transformer. What is the characteristic impedance of the quarter-wave transformer?

28. A shorted coaxial line is used as a resonator. The first resonator is determined to be a parallel resonance and is at 1 GHz.

(a) Draw the lumped-element equivalent circuit of the resonator.

(b) What is the electrical length of the resonator?

(c) What is the impedance looking into the line at resonance?

(d) If the resonator is $\lambda/4$ longer, what is the impedance of the resonator now?

29. A quarter-wave transformer is to be used to match a load of 50 $\Omega$ to a generator with a Thevenin equivalent impedance of 75 $\Omega$. What is the characteristic impedance of the quarter-wave transformer?

30. A coaxial transmission line is filled with lossy material with a relative dielectric constant of $5 - j0.2$. If the line was airfilled it would have a characteristic impedance of 100 $\Omega$.

(a) What is the characteristic impedance of the dielectric-filled line?

(b) What is the propagation constant at 500 MHz?

(c) What is the input impedance of the line if it has an electrical length of 280$^\circ$ and is terminated in a 35 $\Omega$ resistor?

(d) What is the input impedance of the line if it has an electrical length of 280$^\circ$ and is terminated in an inductor of impedance $j35$ $\Omega$?
(e) What is the input impedance of the line if it is 1 km long? Use reasonable approximations. [Hint: Does it matter what the termination is?]

31. A coaxial line is filled with a very slightly lossy material with a relative dielectric constant of 5. The line would have a characteristic impedance of 100 Ω if it was airfilled.

(a) What is the characteristic impedance of the dielectric-filled line?
(b) What is the propagation constant at 500 MHz? Use the fact that the velocity of an EM wave in a lossless air-filled line is the same as that of free-space propagation in air. That is, the propagation constant is the same.
(c) What is the input impedance of the line if it has an electrical length of 90° and it is terminated in a 35 Ω resistor?
(d) At what frequency will the first transverse resonance occur?
(e) At what frequency will the first higher-order stripline mode occur?
(f) At what frequency will the first parallel-plate waveguide mode occur? Do not consider the mode with no field variation, as this cannot be excited.
(g) Identify the useful operating frequency range of the stripline.

32. The strip of a microstrip has a width of 250 µm and is fabricated on a lossless substrate that is 500 µm thick and has a relative permittivity of 2.3. [Parallels Example 4.19 on Page 225]

(a) What is the effective permittivity of the line?
(b) What is the characteristic impedance?
(c) What is the propagation constant at 3 GHz ignoring any losses?
(d) If the strip has resistance of 0.5 Ω/cm and the ground plane resistance can be ignored, what is the attenuation constant at 3 GHz?

33. The strip of a symmetrical stripline has a width of 200 µm and is embedded in a lossless medium that is 400 µm thick and has a relative permittivity of 13, thus the separation, h, from the strip to each of the ground planes is 200 µm.

(a) Draw the effective waveguide model of stripline with magnetic walls and an effective strip width, w_{eff}, which will be approximately the same as with microstrip.
(b) What is the effective relative permittivity of the stripline waveguide model?
(c) What is w_{eff}?
(d) At what frequency will the first transverse resonance occur?
(e) At what frequency will the first higher-order stripline mode occur?
(f) At what frequency will the first parallel-plate waveguide mode occur? Do not consider the mode with no field variation, as this cannot be excited.
(g) Identify the useful operating frequency range of the stripline.

34. Design a microstrip line to have a characteristic impedance of 65 Ω at 5 GHz. The microstrip is to be constructed on a substrate that is 635 µm thick with a relative dielectric constant of 9.8. Ignore the thickness of the strip. [Parallels Example 4.21 on Page 231]

(a) What is the width of the line?
(b) What is the effective permittivity of the line?

35. A load has a reflection coefficient of 0.5 when referred to 50 Ω. If the load is placed at the end of a transmission line with a 100 Ω characteristic impedance.

(a) What is the complex ratio of the forward-traveling wave to the backward-traveling wave on the 100 Ω line at the load end of the line?
(b) What is the VSWR on the 100 Ω line?
(c) Now consider that the line has a characteristic impedance of 50 Ω. If the line has an electrical length of 45°, what is the reflection coefficient calculated at the input of the line?
(d) What is the VSWR on the 50 Ω line?

36. Design a microstrip shorted stub at 10 GHz with the following characteristics:
   - Characteristic impedance of 60 Ω.
   - A substrate with a relative permittivity of 9.6 and thickness of 500 µm.
• Input impedance that is a reactance of $\jmath 60 \Omega$.

(a) What is the width of the microstrip line?
(b) What is the length of the line in centimeters?
(c) What is the effective permittivity of the line?
(d) If the line is a one-quarter wavelength longer than that calculated in (b), what will the input reactance be?
(e) Regardless of your calculations above, what is the input admittance of a one-quarter wavelength long shorted stub?

37. A load has an impedance $Z = 75 + \jmath 15 \Omega$.

(a) What is the load reflection coefficient, $\Gamma_L$, if the system reference impedance is 75 $\Omega$?
(b) Design a stub at the load that will make the impedance of the load plus the stub, call this $Z_1$, purely real; that is, the reflection coefficient of the effective load, $\Gamma_L$, has zero phase. Choose a stub characteristic impedance of 75 $\Omega$. (Design specifications require complete electrical information such as whether the stub is open- or short-circuited, and the electrical length of the stub.)
(c) Design a quarter-wave transformer that will present a matched termination to a source with a system reference impedance of 50 $\Omega$. (The design must include full electrical specifications such as the characteristic impedance of the transmission line and its electrical length. The structure is the source, a $\lambda/4$ transformer, a stub, and the load.)
(d) Now convert the electrical specifications of the design into a physical specification. Assuming that the transmission line technology to be used is a microstrip line and the substrate medium is fixed with the following parameters: frequency $f = 1$ GHz, substrate thickness $h = 0.5$ mm, substrate relative dielectric constant $\varepsilon_r = 10$. You must design the widths and lengths of the stub and the quarter-wave transformers.

38. Design a microstrip line to have a characteristic impedance of 20 $\Omega$. The microstrip is to be constructed on a substrate that is 1 mm thick with a relative dielectric constant of 12. [Parallels Example 4.21 on Page 231]

(a) What is the width of the line? Ignore the thickness of the strip and frequency effects.
(b) What is the effective permittivity of the line?