### 3.6 MULTIPATH PROPAGATION

A radio signal spreads out in different directions as it radiates away from the broadcast antenna. Parts of the spreading wave will encounter reflecting surfaces, and the wave will scatter off these objects. In an urban environment, the wave might reflect off buildings, moving trains, or airplanes.

Multipath occurs when a signal takes two or more paths from the transmitting antenna to the receiving antenna. We'll assume that one signal, the direct ray, travels directly from the transmitter to the receiver. The direct ray is usually (but not always) the strongest signal present in the receiving antenna.

The other signals (or rays) arrive at the receiving antenna via more roundabout paths. These reflected signals eventually find their way to the receiving antenna. In our analysis, we'll assume these indirect rays arrive after the direct ray and that the indirect rays are weaker in power than the direct rays.

**FIGURE 3-11** In a severe multipath environment, a signal might bounce off several different reflectors before it arrives at a receiver. The result can be severe distortion.
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The equation for the direct signal is

\[ V_D(t) = \cos(\omega_0 t) \]  

(3.18)

where

\[ \omega_0 = 2\pi f_0 \text{ = the angular operating frequency} \]

We’ll assume the amplitude of the direct ray is unity and the phase angle is zero. The equations for the reflected rays are

\[ V_{R,1} = \rho_1 \cos(\omega_0 (t - \tau_1)) \]
\[ V_{R,2} = \rho_2 \cos(\omega_0 (t - \tau_2)) \]
\[ \vdots \]
\[ V_{R,n} = \rho_n \cos(\omega_0 (t - \tau_n)) \]

(3.19)

where

\[ \rho_n = \text{a real number describing the difference in amplitude between the direct ray and the } n\text{-th reflected ray} \]
\[ \tau_n = \text{the time difference of arrival between the direct and the } n\text{-th reflected ray. The } n\text{-th ray arrives } \tau_n \text{ seconds after the direct ray.} \]

We may find it convenient to rewrite the equations of (3.19) as

\[ V_{R,1} = \rho_1 \cos(\omega_0 t - \phi_1) \]
\[ V_{R,2} = \rho_2 \cos(\omega_0 t - \phi_2) \]
\[ \vdots \]
\[ V_{R,n} = \rho_n \cos(\omega_0 t - \phi_n) \]

(3.20)

where

\[ \phi_1 = -\omega_0 \tau_1 \]
\[ \phi_2 = -\omega_0 \tau_2 \]
\[ \vdots \]
\[ \phi_n = -\omega_0 \tau_n \]

(3.21)

The scattering mechanism can be more complex than this simple description. Often, a wave will experience a phase shift, polarization change, or some other change when it encounters a scattering surface. For our analysis, we assume that the direct and reflected waves have different amplitudes (described by \( \rho_n \)) and that the waves arrive at the receiving antenna at slightly different times (described by \( \tau_n \)).

The characteristics of multipath channels usually change over time because the geometry of the channel changes. Consider a wireless phone operating in an office environment. The propagation environment contains all sorts of reflectors (e.g., file cabinets, desks, doors, Venetian blinds). As the signal encounters each of these objects, a reflection occurs. The net channel characteristics arise from the sum of all these individual channels. For example, when the user leans back in his chair or begins pacing excitedly during a difficult conversation, the electrical lengths of all the paths change simultaneously. This changes both \( \rho_n \) and \( \tau_n \) for every channel. Even if the user remains perfectly still, the
channel will change as people pull out file cabinet drawers or open and close doors. Small differences in the arrival time of the different signal can make a big difference in the received signal quality. Since the propagation characteristics are continually changing, we will eventually describe both $\rho_n$ and $\tau_n$ of equation (3.19) in statistical terms.

### 3.6.1 Two-Ray Analysis

The two-ray multipath analysis assumes that only two rays are present at the receive antenna: the direct ray $V_D(t)$ and a single reflected ray $V_{R,1}(t)$. The two signals present at the receive antenna are

$$V_D(t) = \cos(\omega_0 t) \quad (3.22)$$

and

$$V_{R,1} = \rho_1 \cos[\omega_0(t - \tau_1)] \quad (3.23)$$

We can rewrite equation (3.23) as

$$V_{R,1} = \rho_1 \cos[\omega_0 t - \omega_0 \tau_1] = \rho_1 \cos[\omega_0 t + \phi_1] \quad (3.24)$$

where

$$\phi_1 = -\omega_0 \tau_1 \quad (3.25)$$

Our random variables are $\rho_1$ and $\phi_1$. The complete signal present at the receive antenna is

$$V_{Rx}(t) = V_D(t) + V_{R,1}(t) = \cos(\omega_0 t) + \rho_1 \cos(\omega_0 t + \phi_1) \quad (3.26)$$

We’ll experience a multipath fading event when the received signal power falls below some arbitrary threshold. Analysis of equation (3.26) reveals

$$V_{Rx}(t) = 0 \text{ whenever } \begin{cases} \rho_1 = 1 \\ \phi_1 = 180^\circ \end{cases} \quad (3.27)$$

Both of the $\rho_1$ and $\phi_1$ conditions must be true to observe a fading event. Most of our analytical effort will be spent examining equation (3.26) when $\rho_1$ is in the neighborhood of unity and $\phi_1$ is in the neighborhood of $180^\circ$.

As Figure 3.12 shows, we can interpret equation (3.26) as the addition of two phasors. The direct ray is represented by a phasor of unity length at $0^\circ$, whereas the indirect ray is...
FIGURE 3-13 = The phasor interpretation of equation (3.26) for various values of \( \rho_1 \).

FIGURE 3-14 = Phasor analysis of two-ray multipath.

represented by a phasor of length \( \rho_1 \) at an angle of \( \phi_1 \). Figure 3-12 also shows the locus of the resultant vector as the phase of the indirect ray varies over \( 0^\circ \) to \( 360^\circ \).

Figure 3-13 shows the phasor diagrams for several values of \( \rho_1 \). The resultant vector \( V_{Rr}(t) \) almost equals the direct ray when \( \rho_1 \) is very small. As a result, the receiver sees a nearly undistorted signal. As \( \rho_1 \) approaches unity, the phase and amplitude of \( V_{Rr}(t) \) change wildly with the phase of the indirect ray. Figure 3-14 formalizes the variables of Figure 3-12 and Figure 3-13.

Vector analysis of Figure 3-14 allows us to rewrite equation (3.26) as a single cosine with an amplitude and phase change

\[
V_{Rr}(t) = \cos(\omega_0 t) + \rho_1 \cos(\omega_0 t + \phi_1) = \beta_1 \cos(\omega_0 t + \theta_1) \tag{3.28}
\]

where

\[
\beta_1^2 = 1 + 2 \rho_1 \cos(\phi_1) + \rho_1^2 \quad \text{and} \quad \theta_1 = \tan^{-1} \left( \frac{\rho_1 \sin(\phi_1)}{1 + \rho_1 \cos(\phi_1)} \right) \tag{3.29}
\]
The ratio of the power in $V_{R}(t)$ to the power in the original cosine wave is $\beta^2$. The change in phase between the original cosine wave and $V_{R}(t)$ is $\theta_1$.

Figure 3-15 shows the power and phase shift of the received signal $V_{R}(t)$ with respect to the direct ray for various values of $\rho_1$ and $\phi_1$. As expected, the worst signal attenuation occurs when $\rho_1$ is unity and $\phi_1$ is $180^\circ$. The received signal is completely canceled under these conditions. The nulls are deep and sharp when $\rho_1$ is close to unity. A small change in $\theta_1$ can result in a large change in received signal strength. We see an increase in received signal strength whenever

$$-90^\circ \leq \phi_1 \leq 90^\circ$$

(3.30)

The maximum increase is only 6 dB (when $\rho_1$ is unity and $\phi_1$ is $0^\circ$). Unfortunately, the same $\rho_1$ that produces the maximum increase in signal power also produces the worst attenuation. The only difference is the phase $\phi_1$.

Multipath affects the phase of the received signal as well as its amplitude. When $\rho_1$ is in the neighborhood of unity, the received phase changes abruptly when $\phi_1$ passes through $180^\circ$.

As you might expect, the sudden phase change plays havoc with phase-encoded signals.

**EXAMPLE**

**Multipath Fading**

You've probably experienced multipath fading without even knowing it. You pull your car up to a stoplight and the FM radio fades out, but you find you can bring the signal back in by scooting your car forward a few feet.

When the FM signal died, two (or more) signals from the transmitter were arriving at your car exactly out of phase, and they canceled. Moving the car changed the phase difference between the arriving signals by a small amount, and, as Figure 3-15 shows, that can result in a large change in the received signal strength.
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EXAMPLE

Multipath Fading

The author once lived along the flight path to an airport. Generally, the television reception was excellent, except when an airplane flew along one particular flight path. The television reception faded in and out as the airplane moved. The rate of change was slow at first, taking perhaps 3 seconds to change from perfect to a completely unwatchable.

As the airplane continued along its approach toward the airport, the rate of change of the signal strength increased until the picture faded in and out several times a second. Finally, as the geometry continued to change, the television reception cycle time gradually decreased back to its 3 second fade-in–fade-out behavior. This entire interference event lasted perhaps 20 seconds from start to finish.

A commercial airliner is an excellent signal reflector, so the signal reflected from the airplane was about as strong as the signal directly from the television transmitter ($\rho_1$ was about equal to unity). As the airplane moved along its flight path, the changing geometry caused the signals arriving at the receive antenna to add up alternately in phase and out of phase.

3.6.2 Frequency Response

We can describe the effects of multipath in the frequency domain. Equations (3.28) and (3.29) describe the two signals present at our receiving antenna. We’re interested in the relationship among $f_0$, $\tau_1$, $\beta_1^2$, and $\phi_1$. Rewriting equation (3.29) in terms of differential time delay, $\tau_1$, and frequency of operation, $\omega_0$, produces

$$\beta_1^2(\omega_0) = 1 + 2\rho_1 \cos(\omega_0 \tau_1) + \rho_1^2$$

and

$$\theta_1(\omega_0) = \tan^{-1}\left[\frac{-\rho_1 \sin(\omega_0 \tau_1)}{1 + \rho_1 \cos(\omega_0 \tau_1)}\right]$$

where $\tau_1$ and $\rho_1$ are fixed, and $\omega_0$ is the variable. Figure 3-16 shows the amplitude characteristics of multipath in the frequency domain. These graphs show the frequency response of a channel consisting of a direct ray and a reflected ray delayed by $0.2$ nsec, $0.5$ nsec, $1.0$ nsec, and $2.0$ nsec. This figure makes it obvious why this effect is sometimes called frequency-selective multipath. Figure 3-17 shows the phase response of the multipath channel.

FIGURE 3-16

Two-ray multipath interpreted as a filter. The amplitude response of a multipath channel for various values of differential path delay $\tau$. 

Frequency-Selective Multipath - (Various $\tau$’s)
Light travels about 1 foot per nsec, so the differential path delays of Figure 3-16 represent differential path lengths of only 2 feet in the worst case.

This simple, two-ray multipath channel acts like a *comb* filter. That is, the channel exhibits regularly spaced intervals of high attenuation, making the transfer function resemble the teeth of a comb. The frequencies of the nulls are

$$f_{null} = \frac{2n + 1}{2\tau_1}, \quad n = 0, 1, 2 \ldots$$  \hfill (3.32)

Channels with large differential time delays (i.e., large values of $\tau$) produce nulls that are very close together in frequency. Large differential delay produces closely spaced nulls in the frequency response.

In high-frequency (HF) communications, we often bounce the signal off the ionosphere to achieve over-the-horizon communications. The signal can take several paths from the transmitter to the receiver, and the result is that the differential time delay can be on the order of milliseconds, which produces notches separated by only hundreds of hertz. As $\rho_1$ approaches unity, the more widely the channel characteristics vary with changing time delay.

**EXAMPLE**

**Frequency of the Multipath Notches**

Derive equation (3.32).

**Solution**

Equation (3.25) relates the differential time delay $\tau_1$ to the phase between the direct and reflected rays. We will experience a null whenever $\phi_1$ is an odd multiple of $\pi$ so we can write

$$\phi_1 = -\omega_0 \tau_1$$

$$2(n + 1)\pi = -2\pi f_{null} \tau_1$$

$$f_{null} = \frac{2n + 1}{2\tau_1}$$  \hfill (3.33)


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3.6.3 Three-Ray Analysis

Let’s extend our two-ray analysis of multipath into a three-ray model. The direct ray is given by

\[ V_D(t) = \cos(\omega_0 t) \]

(3.34)

and the two reflected rays are

\[ V_{R,1} = \rho_1 \cos[\omega_0(t - \tau_1)] \]

and

\[ V_{R,2} = \rho_2 \cos[\omega_0(t - \tau_2)] \]

(3.35)

The complete signal, as seen by the receiving antenna, is

\[ V_{Rx}(t) = \cos(\omega_0 t) + \rho_1 \cos[\omega_0(t - \tau_1)] + \rho_2 \cos[\omega_0(t - \tau_2)] \]

(3.36)

where

\[ \phi_1 = -\omega_0 \tau_1 \]

and

\[ \phi_2 = -\omega_0 \tau_2 \]

(3.37)

As in the two-ray analysis, we can rewrite equation (3.36) as

\[ V_{Rx}(t) = \beta_2 \cos(\omega_0 t + \theta_2) \]

(3.38)

where

\[ \beta_2^2 = 1 + \rho_1^2 + \rho_2^2 + 2 \rho_1 \cos(\phi_1) + 2 \rho_2 \cos(\phi_2) + 2 \rho_1 \rho_2 [\cos(\phi_1) \cos(\phi_2) + \sin(\phi_1) \sin(\phi_2)] \]

(3.39)

and

\[ \theta_2 = \tan^{-1} \left[ \frac{\rho_1 \sin(\phi_1) + \rho_2 \sin(\phi_2)}{\rho_1 \cos(\phi_1) + \rho_2 \cos(\phi_2)} \right] \]

(3.40)

The general analysis of the three-ray problem is not as useful as the two-ray case, but specific examples are helpful. Figure 3-18 shows the amplitude and phase responses for a three-ray channel with the arbitrary values \( \rho_1 = 0.85 \), \( \tau_1 = 0.95 \) nsec, \( \rho_2 = 0.50 \), and \( \tau_2 = 2.2 \) nsec. Even this simple three-ray model produces many amplitude and phase perturbations, indicating the complexity of a more complicated environment.

3.6.4 n-Ray Analysis

We can easily extend the three-ray multipath model to an \( n \)-ray model. We assume that \( n \) separate signals arrive at the receiving antenna, each with its own \( \rho \) and \( \tau \). We will experience a multipath fading event whenever the sum of the direct signal and the \( n \) reflected signals add to zero.

3.6.4.1 Indoor Multipath

The \( n \)-ray model agrees closely with experimentally observed propagation over indoor channels. The receiving antenna collects many different rays from different directions, and most of the received signals exhibit about the same magnitude. These conditions combine to produce a rapidly fluctuating RF environment. Theory and measurements reveal that, under these conditions, the probability density function of the fade statistics is
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Ricean. The properties of a Ricean distribution cause the received signal strength to vary widely and rapidly for a large percentage of the time.

3.6.5 Spatial Redistribution of Energy

If there was only one ray present in the environment, the energy available in that ray would be equally distributed over three-dimensional space (three space), and the received signal strength would not depend on the physical location of the antenna. Strong multipath propagation has the effect of redistributing the signal energy over three space. Figure 3-19 through Figure 3-22 show how multipath affects the physical distribution of received energy.

![Signal Energy Distribution](image)

**FIGURE 3-19** Two-ray multipath energy distribution across an aperture (or wall). The direct ray \( (\rho_0 = 1) \) is arriving perpendicular to the wall \( (az_0 = 0^\circ, el_0 = 0^\circ) \), while the single indirect ray \( (\rho_1 = 0.95) \) is arriving at an azimuth of \( 10^\circ \) and an elevation of \( 0^\circ \) \( (az_1 = 10^\circ, el_1 = 0^\circ) \). Signal maxima are shown in red, and signal minima are shown as blue. The distance between maxima is determined by the frequency \( (6 \text{ GHz here}) \) and the angle of arrival of the indirect ray.
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Signal Energy Distribution @ 6.00 GHz, $\lambda = 0.050$ m

$\rho_0(1.000), az_0(0.0), el_0(0.0)$
$\rho_1(0.950), az_1(10.0), el_1(-5.0)$;

**FIGURE 3-20** Two-ray multipath energy distribution across an aperture. This is the same multipath environment as shown in Figure 3-19 except that the indirect ray is arriving from an azimuth of $10^\circ$ and an elevation of $-5^\circ$ ($az_1 = 10^\circ$, $el_1 = -5^\circ$). The vertical minima of Figure 3-19 have changed from perfectly vertical to sloped.

**FIGURE 3-21** Three-ray multipath energy distribution across an aperture. This is the same multipath environment as shown in Figure 3-20 with the addition of a second indirect multipath component at ($\rho_2 = 0.51$, $az_2 = -5^\circ$ and $el_2 = 7.2^\circ$).

Figure 3-23 shows the geometry when two coherent signals arrive at the same physical space from different directions. The signals are plane waves with different path delays, and they have about the same magnitude. The multipath produces areas of high signal strength (where two lines intersect) and areas of little or no signal (between the signal maxima).

In Figure 3-23, $\theta_a$ is the angle between the two signals, and the distance between maxima (and the distance between minima) is

$$d_{Max,Max} = d_{Min,Min} = \frac{\cos\left(\frac{\theta_a}{2}\right)}{\sin(\theta_a)} = \frac{\lambda}{\theta_a} \quad \text{for small } \theta_a$$

(3.41)
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FIGURE 3-22  The three-ray multipath distribution of Figure 3-21 viewed from two different positions. The left figure shows the energy distribution directly at the wall. The right figure shows the energy distribution measured 1.75 m behind the wall. Note that the energy null at (0.25 m, 1.1 m) in the left figure is an energy maximum in the right figure.

FIGURE 3-23  The geometry of a two-ray multipath environment. The lines represent the signal peaks and are separated by the wavelength $\lambda$.

The distance the maxima and minima is half the distance between maxima, so we can write

$$d_{\text{Max,Min}} = \frac{d_{\text{Max,Max}}}{2} = \frac{\lambda \cos \left( \frac{\theta_a}{2} \right)}{2 \sin(\theta_a)}$$

(3.42)
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EXAMPLE

Multipath Maxima and Minima

Assume that two multipath waves arrive with a $\theta_a$ of 3°. If the operating frequency is 4.5 GHz, find the distance between the multipath maxima.

Solution

The wavelength at 4.5 GHz is 0.0667 meters. The angle of arrival is $3^\circ = 0.0524$ radians. Using equation (3.41), we can write

$$d_{\text{Max,Max}} = \frac{\lambda \cos \left(\frac{\theta_a}{2}\right)}{\sin (\theta_a)}$$

$$= (0.0667) \frac{\cos \left(\frac{3^\circ}{2}\right)}{\sin (3^\circ)}$$

$$= 1.27 \text{ meters}$$

(3.43)

3.6.6 Multipath Behavior over Time

In the two-ray multipath model, the worst-case situation occurs when $\rho_1$ is near unity. As the phase varies, the received signal power varies from zero (total cancellation) to four times the power available from a single signal (for in-phase addition).

An environment that reflects signals with little loss is conducive to multipath. The multipath environment over water is normally very bad because the water, being conductive, reflects signals with little loss. Consequently, the multipath signals arriving at the receiving antenna generally exhibit $\rho$’s that are very close to unity. The wave motion of the water causes the phase and magnitude of the reflected signal to vary rapidly. This causes the sum of the direct and reflected rays to vary rapidly.

Urban environments are notorious multipath environments. This is especially true in areas that include tall, mirrored buildings and other metallic structures. The multipath in a city usually isn’t as dynamic as it is over water, but it still varies significantly over time.

The situation is very bad inside commercial office buildings. Consider a signal traveling from a transmitting antenna to a receiving antenna—let’s use a 900 MHz cordless telephone as an example. The wavelength at 900 MHz is about 33 cm. So, when the difference in path lengths changes by half of a wavelength or 16.5 cm, the received signal strength has the opportunity to vary between +6 dB and –30 dB relative to the strength of the direct signal. If the user of the cordless telephone is agitated and begins to pace about during the conversation, the received signal strength will vary dramatically. The wavelength shortens as the frequency increases and the effect gets worse, and we experience more signal dropouts per unit time.

We must ask questions such as:

- How often will I experience a fade of 20 dB?
- How long will the fade last?
- How many minutes per month will my received signal strength be below a particular value?
### 3.6.7 Multipath Statistics

We are forced to describe multipath effects statistically, and we must determine the characteristics of the statistics that govern the values of $\rho_n$ and $\tau_n$ of equation (3.24). We begin by assuming that both $\rho_n$ and $\tau_n$ follow a Gaussian distribution. We can write the equation for the reflected ray in two equivalent forms:

$$V_{R,1} = \rho_1 \cos \left[ \omega_0 (t - \tau_1) \right]$$

and

$$V_{R,1} = \rho_1 \cos \left[ \omega_0 t + \phi_1 \right] \quad (3.44)$$

where

$$\phi_1 = -\omega_0 \tau_1 \quad (3.45)$$

What are the statistics that describe $\theta_1$ if we assume that $\tau_1$ is a Gaussian-distributed random variable? We find that, if the variation in the time delay, $\Delta \tau_n$, is much greater than the carrier signal period or

$$\Delta \tau_n \ll \frac{1}{f_0} \quad (3.46)$$

the angle $\theta_1$ will be uniformly distributed. All angles between 0.0° and 359.999...° are equally likely. We can show that a Gaussian-distributed $\rho_1$ and a uniformly distributed $\theta_1$ will combine to produce a resultant whose length is Rayleigh distributed. The Rayleigh distribution tells us that the received signal strength is usually constant but will occasionally suffer a deep fade.

The statistics of $\rho_1$ and $\tau_1$ also govern the multipath fade event time. Rather loosely, a multipath fade event time is the average amount of time the signal strength falls below a given level. For a system that can withstand fading events of 30 dB, we are interested in the percentage of time our received signal strength will be attenuated by 30 dB or more. We are also interested in the average amount of time that a single fading event will last (i.e., how long will it be from the time the signal drops below the 30 dB level to the time it finally rises above the 30 dB level). This information is will be important when we discuss multipath mitigation.

### 3.6.8 Multipath Mitigation

Multipath fading does not reduce the average energy of the received signal. The signal energy is redistributed over time and space. If we design our communications links to be insensitive to this energy redistribution, then our link can approach the theoretical performance of an additive white Gaussian noise (AWGN) channel.

Generally, all multipath mitigation techniques use some form of diversity. The transmitter sends sufficient information over two or more statistically separate channels. The receiving antenna collects this energy, and, by careful manipulation, it can recover the transmitter’s data. The statistically separate channels can include frequency diversity, time diversity (using data coding and interleaving), and space diversity.

#### 3.6.8.1 Antenna Pattern

If we limit the number of signals our antenna views, we will limit the probability of receiving two separate signals from the same transmitter. We can satisfy this requirement with a narrow-beamwidth, low-sidelobe antenna.

Sometimes we can’t control the antenna, however. Consider the case of a cellular telephone. The user may be in an arbitrary location and may move rapidly during a transmission. We must use an omnidirectional antenna to satisfy the user’s needs. As
we’ve seen, omnidirectional antennas are not helpful in a multipath environment because they pick up signals from every direction.

### 3.6.8.2 Frequency Diversity

A signal experiences a fade as it travels from the transmit antenna to the receive antenna because the direct and the reflected signals cancel out. The cancellation occurs when the difference in path lengths between the direct and reflected rays is an odd multiple of 180°. The length of the transmission paths in terms of wavelengths determines the phase of the two arriving signals.

For example, imagine a system that is exhibiting a deep fading event. The magnitude of the received signal is almost zero because the phase between the direct and reflected rays is almost 180°. Now, imagine we change the frequency slightly. Two signals traveling from the same transmit antenna to the same receive antenna will probably travel over the same path if the frequencies of the signals are nearly identical. A signal at frequency $f_1$ will probably travel over the same path as a signal at $f_2$ if $f_1 \approx f_2$.

Although the physical paths taken by the signals at $f_1$ and $f_2$ are about the same, the path lengths (in wavelength units) are different. At $f_1$, the path length might be 143,445.278 wavelengths for the direct path and 144,322.778 wavelengths for the reflected path (note that these two path lengths differ by 180° so we are experiencing a fade). Conversely, at $f_2$, the same physical paths might translate into 142,227.777 wavelengths (direct path) and 142,990.101 wavelengths. Since the phase difference between the direct and reflected signals is no longer 180°, the signal is not in a fade at this frequency.

The instantaneous fading behavior of a channel is a strong function of frequency. If we experience a fade at one frequency, the channel may not be fading at a slightly different frequency. If we transmit the same signal at two frequencies at the same time, we can take advantage of this behavior. When the signal fades on one frequency, there is a good possibility that the signal on the other frequency might not be fading. There is no guarantee, however. It’s entirely possible that both channels will be in fade at the same time. All we’ve really done is to change the fading statistics in our favor.

Studies by the telephone companies indicate that the two frequencies should be separated by at least 2% of the center frequency or by 20 MHz, whichever is greater. Most microwave telephone relay systems that use frequency diversity have separations of 2–5% of the lower frequency. Figure 3-24 shows the received signal power of two channels separated in frequency by 4% of center frequency.

Frequency diversity is effective, but it isn’t often used because it wastes spectrum. Fortunately, there are other multipath mitigation methods.

### 3.6.8.3 Spatial Diversity

We’ve seen that a multipath fade will occur when the geometry among the transmitting antenna, the receiving antenna, and some reflector produces a 180° phase shift between the direct and delayed signals. If the geometry changes just slightly, the phase relationship between the two received signals changes, and we often see a dramatic increase in received signal strength.

We can use this effect to change the multipath statistics in our favor as Figure 3-25 shows. We can use one transmitting antenna and two, physically separate receiving antennas and allow the receiving equipment to choose the best signal from the two antennas. If the signal from one antenna experiences a fade, the signal from the second antenna might not be in a fade. As with frequency diversity, it’s possible that the signals from both receive antennas are in a fade.
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FIGURE 3-24 A recording of frequency diversity transmission. The two signals followed identical paths were spaced 1,080 MHz apart (4% of the center frequency). Note that deep fades do not often occur simultaneously.

FIGURE 3-25 A diagram of a home wireless local area network (WLAN) transceiver. The access point on the left will often use two antennas set at different angles to use path and polarization diversity.

**EXAMPLE**

**Space Diversity**

A particular WLAN access point has two antennas physically separated by 4 inches. How many wavelengths separate the two receive antennas?

**Solution**

Four inches is about 10.1 cm. The wavelength at 2.4 GHz is

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ meters/sec}}{2.4 \times 10^9 \text{/sec}} = 0.125 \text{ meters} = 12.5 \text{ cm} \]  

(3.47)

So 4 inches is about 1 wavelength. Experience indicates that you will still achieve significant diversity improvement even when the receive antennas are this close together. The relatively small antenna separation will still improve transmission reliability.
3.6.8.4 Polarization Diversity

Vertically polarized waves tend to reflect off vertically oriented objects, whereas horizontally polarized waves tend to reflect off horizontally oriented objects. Consider a window covered by horizontal Venetian blinds. Assume the blinds are in the open position so only a small amount of metal is presented to the oncoming waves. Vertically polarized waves will pass through the blinds with little attenuation, whereas horizontally polarized waves will be almost completely reflected. From a multipath perspective, the physical paths taken by vertically polarized waves will likely be different from the path taken by horizontally polarized waves. If we're experiencing a fade on one polarization, we may not be fading on the other polarization. This is polarization diversity. Studies have suggested that polarization diversity is as effective as space or frequency diversity for multipath protection.

Polarization diversity does present some practical problems. Both the transmitter and receiver require dual-polarization antennas, which can be physically cumbersome.

3.6.8.5 Time Diversity

We can also use time diversity as a weapon against multipath fading. Time diversity is useful mostly when we are transmitting digital data and when we’re using forward error correction (FEC).

Once we determine the fading statistics, we can determine how long an average fade will last. For example, if our system can endure a 10 dB reduction in signal strength, we want to know how long the signal will be attenuated by 10 dB or more in a typical fading event. The fade will create a burst of errors, which lasts for the duration of the multipath event. When we know the statistics of a fading event (i.e., we know how long an average fading event will last, and we know the average number of fades per second), we also know how many bits per second we will be unable to decode. We can now design a FEC scheme to correct these data errors.

Finally, we find that we must spread our data out over time to take the best advantage of the FEC and the error statistics. For example, if we experience a fade long enough to obliterate both the data bits and the FEC bits, then that data are lost. We can spread both the data and their associated FEC bits over time so that an average multipath event can’t generate unrecoverable errors.

War Story

A form of time diversity is used in audio compact discs. The data are spread out over a large area of the disk. If one section of the disk is damaged or covered by dirt, then the error correction circuitry in the CD player can correct the errors as long as it can also read data from the clean areas of the disk.

3.6.9 Multipath Equalizers

We can view multipath as a time-varying filter between the transmitter and receiver that distorts the signal. Multipath equalizers are filters used to mitigate the effects of multipath propagation on the received signal by dynamically adapting to the changing multipath channel.

One method to measure our channel is to observe the output of the demodulator and adaptively make adjustments to the equalizing filter. This is adaptive, decision-based
equalization. A second method is to place a known piece of data in the transmitted signal, that is, place a pre-, mid-, or post-amble in the signal at regular intervals. We examine the \textit{-amble} at the receiver and use that data to configure the equalizing filter.

\section*{3.7 BIBLIOGRAPHY}


\section*{3.8 PROBLEMS}

1. **Cell Coverage**  Consider the coverage of a cellular telephone tower under the assumption of propagation through a homogenous medium with $n = 3.4$ (this assumption is wildly inaccurate but for the sake of a homework problem). If the path loss from the cell to any user should not be greater than $A_{\text{dB}}$, find an expression for the radius of the cell in terms of $A_{\text{dB}}$ for a frequency of 1,800 MHz.
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2. **Free Space Path Loss**  The free-space path loss equation describes the loss of signal power as the power emitted by an isotropic radiator spreads out over the surface of a sphere. Free-space path loss is often referred to as spreading loss. The free-space path loss equation is

\[
PL_{FS} = \left( \frac{P_{Tx}}{P_{Rx}} \right) = \left( \frac{4\pi D}{\lambda} \right)^2
\]

where
- \(d\) = the distance between the transmitting and receiving antennas
- \(\lambda\) = the wavelength

Given this geometrical interpretation of free-space path loss or spreading loss, explain why equation (3.51) contains a reference to wavelength.

3. **Three-Ray Multipath**  Consider a 980 MHz signal experiencing three-ray multipath. The transmitted signal is \(s(t)\). The received signal is

\[
r(t) = A_0 s(t) + A_1 s(t - \tau_1) + A_2 s(t - \tau_2)
\]

where
- \(A_0 = 0.4\)
- \(A_1 = 1.0, \tau_1 = 5.125 \mu\text{sec}\)
- \(A_2 = 0.7, \tau_2 = 8.334 \mu\text{sec}\)

a. What is the distance in meters between the longest and shortest path?
b. If the transmitted signal is a cosine, what are the relative phase angles between the three multipath components?
c. If the transmitted signal is a cosine, are there any values for \(\tau_1\) and \(\tau_2\) that will cause \(r(t)\) to be equal to zero (i.e., infinite attenuation)? If so, give values for \(\tau_1\) and \(\tau_2\) that will cause \(r(t)\) to be zero.

4. **Multipath Statistics**  Consider a situation with two-ray multipath. The received signal is

\[
V_{R_1}(t) = V_D(t) + V_{R,1}(t) = \cos(\omega_0 t) + \rho_1 \cos(\omega_0 t + \phi_1)
\]

where
- \(\rho_1 = 1.0\)
- \(\phi_1\) is a uniformly disturbed random variable whose value lies between 0° and 359.999°...

a. What’s the percentage of time that we would expect the received signal \(V_{R_1}(t)\) to be less than 20 dB below the direct signal \(V_D(t)\)? In other words, how often do we expect the received signal to have faded by more than 20 dB?
b. Repeat your analysis for a 30 dB fade.