BASIC MODULATION MATHEMATICS AND DDS (DIRECT DIGITAL SYNTHESIS) PROVIDE DESIGNERS WITH AN ALL-DIGITAL TECHNIQUE FOR Generating POLAR-ENCODED CARRIER SIGNALS.

Polar modulation is a technique whereby a sinusoidal signal, or carrier, having constant radian frequency \( \omega_c \), is time-varied in both magnitude and phase. You can think of polar modulation as transmitting information that both the magnitude (\( r \)) and the angle (\( \theta \)) of a vector simultaneously carry. (You typically reference \( \theta \) to the positive x axis). This vector’s tail attaches to the origin of the polar-coordinate system, and its head attaches to any other point, \( P \), in the coordinate plane (Figure 1).

You can represent any point, \( P \), in the polar plane by the polar point \( r, \theta \). It should be apparent from Figure 1 that you can easily convert a polar coordinate, \( r, \theta \), to a rectangular coordinate, \( x,y \), by projecting point \( P \) onto the x axis at \( x_1 \) and onto the y axis at \( y_1 \). Doing so allows you to convert the polar coordinate, \( r, \theta \), to the rectangular coordinate, \( x,y \), using equations 1 and 2:

\[
x = r \cdot \cos(\theta); \quad (1)
\]

\[
y = r \cdot \sin(\theta). \quad (2)
\]

It is likewise possible to convert a rectangular coordinate \( (x,y) \) to a polar coordinate \( (r,\theta) \) using equations 3 and 4:

\[
r = \sqrt{x^2 + y^2}; \quad (3)
\]

\[
\theta = \text{atan}(y/x), \quad (4)
\]

where \( \text{atan} \) denotes the inverse-tangent function.

Take care when performing rectangular-to-polar conversions with Equation 4. Most calculators and mathematical-software packages provide an \text{atan}() function with only one argument. For example, in \text{atan}(z), \( z \) is a floating-point number that represents the quotient, \( y/x \), in Equation 4, but using the expression “\( y/x \)” to provide the \( z \) argument for the \text{atan}() function presents two problems. First, when the value of \( x \) is zero or both \( x \) and \( y \) are zero, the value of \( y/x \) is undefined. Second, a single value of \( z \) can represent two angles. For example, \text{atan}(1) typically returns a value of \( \pi/4 \) (0.785 rad). However, \( 5\pi/4 \) (3.927 rad) is an equally valid result.

You accomplish polar modulation by varying the magnitude of a carrier signal over time, the phase angle of a carrier over time, or both. The varying magnitude and phase carry the information to be transmitted. In polar form, this varying magnitude and phase are time variations in the position of point \( P \) in Figure 1. That is, the location of \( P \) varies over time, which amounts to simultaneously varying \( r \) and \( \theta \).

For example, suppose the information to be transmitted is a sinusoid of frequency \( f_0 \) and peak amplitude \( A \). You can represent this relationship on the polar-coordinate system by making point \( P \) follow a circular path. This circle centers on the origin and has a radius of \( A \) units. You realize the frequency component by making point \( P \) complete one revolution around the circle every \( 1/f_0 \) seconds. In this case, the polar variable, \( r \), has no time variation but is fixed at a constant value, \( A \). The polar variable, \( \theta \), on the other hand, varies linearly with time, and its value at any given instant is \( \theta = 2\pi f_0 t \). This type of signal, a rotating vector of

\[
\text{Figure 1}
\]

You can represent any point, \( P \), in the polar plane by the polar point \( r,\theta \), or convert it to a rectangular coordinate, \( x,y \), by projecting point \( P \) onto the x axis at \( x_1 \) and onto the y axis at \( y_1 \).

\[
\text{Figure 2}
\]

The quadrature modulator implements polar modulation by polar-to-rectangular conversion.
constant magnitude, is a phasor. The modulation occurs when you impose the information signal—in this case, a phasor—upon the carrier signal.

The most common method of performing polar modulation uses a quadrature modulator (Figure 2). I(t) and Q(t) are the input signals to the modulator and carry the information to be transmitted. The quadrature modulator implements polar modulation by means of the polar-to-rectangular conversion of equations 1 and 2 (Reference 1). That is, you must convert the information signal from polar form (r,θ) to rectangular form (x,y). You can represent the information signal, which is encoded in the time-varying position of P, in polar form as:

\[ P(t) = (r(t), \theta(t)). \]  

That is, the position of point P is a function of time—and, at any time, t, the location of P is given by the time-dependent values of r and θ (r(t) and θ(t), respectively). Equations 1 and 2 provide the means of converting Equation 5 to time-varying x and y signals as equations 6 and 7 show:

\[ x(t) = r(t) \cdot \cos[\theta(t)]; \]  
\[ y(t) = r(t) \cdot \sin[\theta(t)]. \]  

In Figure 2, input I(t) is identical to x(t) in Equation 6, and Q(t) is identical to y(t) in Equation 7. Therefore, the information to be transmitted is encoded into the I(t) and Q(t) signals and mixed with the carrier signal. You can think of the process of mixing the I(t) and Q(t) signals with the carrier signal as a multiplication process—hence, the multiplier symbols in Figure 2. You add the mixed I(t) and Q(t) signals—hence, the summing symbol in Figure 2—to produce the final output. It produces the form of Equation 8, which is the output signal of the quadrature modulator:

\[ g(t) = I(t) \cdot \cos(\omega_t t) + Q(t) \cdot \sin(\omega_t t). \]  

Substitution for I(t) and Q(t) based on equations 6 and 7 yields Equation 9:

\[ g(t) = \{r(t) \cdot \cos[\theta(t)]\} \cdot \cos(\omega_t t) + \{r(t) \cdot \sin[\theta(t)]\} \cdot \sin(\omega_t t). \]  

However, because the cosine function is an even function, it produces the relationship \( \cos(x) = \cos(-x) \), which allows you to write Equation 13 as:

\[ g = r \cdot \cos(\phi - \theta). \]  

Restoring the time dependency of the variables in Equation 14 yields the output signal of Figure 2:

\[ g(t) = r(t) \cdot \cos\{\omega_t t - \theta(t)\}. \]  

Recall that the input signals in Figure 2, I and Q, are the time-varying rectangular coordinates of the information to be transmitted. Equations 8 through 15 reveal that the output of the quadrature modulator takes the form of a polar equation. That is, the \( \cos(\omega_t t) \) portion of Equation 15 establishes a phasor (Figure 1) that rotates with radian frequency, \( \omega_t \) (the carrier signal). However, the phasor magnitude varies with time as \( r(t) \) prescribes, and its instantaneous angle deviates from \( \omega_t t \) as \( \theta(t) \) prescribes. Therefore, \( r(t) \) and \( \theta(t) \) are the polar-form representation of the original rectangular information that I(t) and Q(t) carry.

This result leads to a direct method of polar modulation (Figure 3). Instead of coding the information to be transmitted into rectangular form, you can apply it in polar form. One advantage of this technique is that it requires only one multiplier.

By inspection, Equation 16 gives the output signal, \( z(t) \), as:

\[ z(t) = r(t) \cdot \cos[\omega_t t - \theta(t)]. \]  

You can see that \( z(t) \) is identical to \( g(t) \), which is the output signal for the quadrature modulator of Figure 2. Hence, you establish the equivalency of the two forms of modulation. Once again, \( r(t) \) is the time-varying amplitude, and \( \theta(t) \) is the time-varying phase of the information to be transmitted.

Once you establish the two methods for generating a polar-modulated signal (Figures 2 and 3), you can explore the common QAM (quadrature-amplitude-modulation) case of polar modulation. The name reflects the usual method by which you implement QAM—the quad-
You can also accomplish QAM using the direct polar-modulation form of Figure 3 by properly controlling the amplitude and the phase of the carrier signal according to Equation 16. You use QAM when you want to transmit data that has the form of a serial bit stream but when the spectrum of the output signal must occupy a relatively narrow bandwidth.

To transmit the QAM data, the “value” of a symbol takes on a unique position in a constellation map. That is, each point in the constellation represents a unique combination of bits in the symbol. Figure 4 shows the constellation for 16 QAM. For example, you could assign the 4-bit combination 0000 to the point at the upper right of Figure 4 and assign the 4-bit combination 0001 to the point that the vector indicates. In like manner, each of the 16 possible values of the symbol gets a unique point in the constellation.

If you use QAM, then the \((x,y)\) coordinate of each constellation point embeds the transmitted information. That is, \(I(t)\) carries the “\(x\)” information, and \(Q(t)\) carries the “\(y\)” information. Alternatively, if you employ direct polar modulation, then the \((r,\theta)\) coordinate of each constellation point embeds the information. The simplicity of the constellation diagram is one reason that it often appears in the collection of literature on the subject of data transmission. However, few texts make it clear that the vector in Figure 4 represents a time-dependent amplitude and phase deviation of the carrier phasor. As such, the vector in Figure 4 must move smoothly from point to point and make no instantaneous jump between points. Instantaneous jumps imply a broad frequency spectrum, which is contrary to the purpose of using QAM as a data-transmission scheme. This technique is pulse shaping (Reference 2).

Once you understand the direct method of polar modulation, you can explore its application in the context of DDS (direct digital synthesis). Note that DDS is a digital method for generating a sinusoidal waveform. DDS plays the role of the oscillator in Figures 2 and 3 and generates the carrier sinusoid. The main advantage of DDS is that you can instantaneously change the frequency of the carrier, an extremely desirable feature in data-transmission schemes that must broadcast packets of data to receivers tuned to different carriers. Instead of having multiple transmitters operating at different carrier frequencies, a single DDS can switch to the desired carriers on demand (Figure 5).

DDS uses an N-bit accumulator, an N-bit latch and adder, to successively sum an N-bit number, \(\theta_n\), at the rate of the system clock, operating at frequency \(F_s\). Incidentally, the literature often refers to \(\theta_0\) as the frequency-tuning, or frequency-control, word. The accumulator has an N-bit latch, and the range of binary numbers that its output can represent is \(0 \leq K \leq 2^N - 1\), where \(K\) is the N-bit binary number at the output of the latch at any time. Therefore, when the value of the accumulator is within the amount \(\theta_0\) of its maximum value of \(2^N - 1\), the next cycle of the system clock causes the accumulator to overflow. That is, only the amount by which the accumulator exceeds \(2^N - 1\) remains in the accumulator. Thus, the value of accumulator output, \(K\), increments by a fixed amount, \(\theta_n\), at the rate of \(F_s\). Hence, the average overflow rate of the accumulator (\(f_c\)) is:

\[
f_c = F_s(\theta_0/2^N).
\]  

For example, if the fractional value, \(\theta_0/2^N\) is 0.113, and \(F_s\) is 100 Hz, then the accumulator overflows at a rate of \(f_c = (100 \text{ Hz})(113/1000) \approx 11.3 \text{ Hz}\). Note that \(\theta_0/2^N\) is always a fraction, because \(\theta_0\) is by definition less than \(2^N\). Now, referring to Figure 5, you see that the numeric output, \(K\), of the accumulator is linearly advancing in value by \(\theta_0\) at a rate of \(F_s\) and overflowing at a rate of \(f_c\) (Equation 17).

Now that you have established the function of the accumulator, consider the function of the trigonometric conversion block. You interpret the input to this block, \(K\), as an angle. The output is a number that corresponds to the cosine or sine of that angle—that is, the value along the \(x\) or the \(y\) axis (Figure 1) that the angle determines. To make this computation, the trigonometric conversion block must map the linear range of the integer, \(K\), of the accumulator generates to the linear range of an angle, \(0 \leq \phi < 2\pi\). The relationship \(\phi = 2\pi K/2^N\) easily accomplishes this task. If a trigonometric conversion block implements the cosine function, then its output value is:

\[
y = \cos(\phi) = \cos(2\pi K/2^N).
\]  

Returning to Figure 5, observe that the
output of the trigonometric converter drives a DAC. Hence, the value of y in Equation 18 must be in the form of an integer that spans a range of $2^N$ (the binary range of a D-bit DAC). The method of mapping the value of y to the binary range of the DAC is unimportant. You must understand that the linearly advancing K values at the input of the trigonometric converter represent linearly advancing angle values. The trigonometric converter converts these values, in turn, to amplitude values. Therefore, the output of the trigonometric converter is a numeric sinusoid with a frequency that is the same as the accumulator-overflow rate (Equation 17).

The final stage of the DDS, the DAC, converts digital samples that the trigonometric-conversion-block produces to an analog signal. Hence, except for the DAC’s conversion to analog, digital means alone produces a sinusoid frequency (Equation 17).

With a basic understanding of DDS, you can easily build a DDS-based polar modulator using the polar modulator form of Figure 3. The DDS replaces the oscillator. You set the oscillator or carrier frequency, $f_c$, by providing the proper value of $\theta_0$ from Equation 17 to the DDS. Building the polar modulator requires only a few small adjustments to the basic DDS (Figure 6).

Once you know the basic operation of the DDS, it should be apparent that you can interpret the output of the accumulator as the instantaneous phase angle of the carrier signal ($\omega_0 t$ in Equation 16). By inserting a subtraction operation between the accumulator and the trigonometric converter, you can effectively implement the “minus $\theta(t)$” part of Equation 16. Furthermore, inserting a multiplier between the trigonometric converter and the DAC scales the output sinusoid by the amount $r(t)$, effectively implementing the $r(t)$ scale factor in Equation 16 and constituting the salient features of a DDS-based polar modulator.

Two caveats are not apparent in the DDS-polar modulator. First, the DDS is a sampled system that operates at the system clock rate, $F_s$. Therefore, the subtraction element and multiplier in Figure 6 must also operate at $F_s$. However, $\theta(t)$ and $r(t)$ in Figure 6 represent the information you want to transmit, which the system samples at a rate that depends on the input data. The problem is that the values of $r(t)$ and $\theta(t)$ occur at the system-clock rate, $F_s$. Hence, you must apply a sample-rate conversion to the symbol samples to match the system-clock rate for processing through the multiplier and the subtraction elements of Figure 6. If you do not employ a sample-rate conversion, the spectral images of the symbol spectrum, which occur in frequency bands that are multiples of the symbol rate, appear in the final output spectrum—an undesirable consequence of sampling theory. The topic of sample-rate conversion is an important aspect of designing a practical DDS-based polar modulator (Reference 3).

The second caveat is that the physical-hardware implementation of the trigonometric-conversion block may exhibit delay. That is, a significant delay may occur between when an input value, $K$, arrives at the trigonometric-conversion block and when the “answer” appears at the output. This delay, latency, can span several system-clock cycles. Why is it such a problem? Recall that a polar coordinate, $r, \theta$, is a coupled pair. The appropriate $r$ must coincide with the appropriate $\theta$ to identify the desired point on the polar plane. If the $r$ sample arrives at the multiplier in Figure 6 before the converted $\theta$ sample, then the coupling of the $r, \theta$ pair breaks, identifying an incorrect polar coordinate. A designer, therefore, must ensure that the delay through the $r(t)$ path matches the delay through the $\theta(t)$ path of Figure 6.

You can implement polar modulation in rectangular form using $x, y$ coordinates or in direct polar form using $r, \theta$ coordinates. You can digitally generate the carrier signal to transmit polar encoded information by means of a DDS. More important, the architecture of the DDS provides an elegant means of implementing a direct polar modulator, which can be less hardware-intensive than a traditional quadrature modulator. Furthermore, you can design an entire polar modulator using digital techniques. Thus, you employ all of digital design’s positive attributes, including stability, repeatability, and low cost, to their full advantage.

References

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