Characterisation of dielectric loaded double ridge waveguides using finite element solvers

The properties of homogeneous single and double ridge waveguides are well understood in the literature [1,2]. The purpose of this paper is to give some calculations on three different dielectric loaded structures. The three topologies investigated here are depicted in Figures 1a, b and c. The computations on the structures in Figures 1a and b are based on a hybrid (Ez/Hz) Finite Element formulation. The functional entering into the description of these problem regions has been dealt with in [3]. The agreement between theory and practice for the arrangement in Figure 1a is in good engineering accord [4]. The calculation on the geometry in Figure 1c is based on a three-component magnetic vector functional [5,8].

The Finite Element Method (FEM) has been widely utilised in the analysis of the cut-off space and propagation constant of dielectric loaded waveguides containing isotropic and anisotropic media. It includes variational expressions based on the two-component (Ez/Hz) hybrid notation [3] and the three-component (magnetic H or electric E) vector formulation [5-8]. The spurious modes encountered in the latter numerical solutions of these types of waveguides may be suppressed by either enforcing the divergence-free constraint in association with tangential and normal boundary conditions [7], or by employing vector elements to ensure only tangential continuity of the field components across element boundaries [8].

The cut-off space of the three lowest quasi LSEm0 modes in double ridge waveguide with a dielectric filler inserted between its ridges has been separately investigated using a mixed modal expansion in conjunction with a transverse field matching technique [9]. A calculation of the normalised propagation constant of one single and one double ridge structure based on a Mixed Spectral Domain Method has separately been employed [10].

The hybrid functional

The problem region tackled in this section consists of a double ridge waveguide with...
a dielectric filler between the ridges. Figures 1a and b indicate the topologies considered here.

A complete solution of this problem region involves the cut-off space, the propagation constant and the field pattern. In an inhomogeneous problem region the latter two quantities have to be calculated at each and every frequency. One solution is to have recourse to a variational technique. This involves constructing an energy functional which when extremised by having recourse to the Rayleigh-Ritz method produces a solution which satisfies the original wave equation [11]. The formulation of the functional for this sort of problem region may be developed in terms of the longitudinal field components \( \frac{E_z}{H_z} \) [3], or the three-component magnetic (\( H \)) or electric (\( E \)) field vectors [5,8]. The former choice has been adopted here for the arrangements in Figures 1a and b and the latter formulation for that in Figure 1c.

The construction of the longitudinal \( \frac{E_z}{H_z} \) functional of a composite problem region containing a number of homogeneous regions (r) starts by premultiplying the vector form of the Helmholtz equation in a typical region by the transpose of the conjugate vector field, integrating the ensuing quadratic equation over the cross-section of the region in question and utilising Green’s theorem in a plane to map a surface integral into a contour one. Application of the appropriate boundary conditions on the contour \( \mathbf{E} \times \mathbf{n} = 0 \) of a homogeneous region of cross-sectional area \( \Omega \) produces the required functional [3].

Extremising with respect to the unconstrained variables \( \{\varepsilon_r\}, \{\eta_0\} \) yields the eigenvalue problem for the overall inhomogeneous region [3]

\[
\begin{align*}
\sum_r \int_\Omega \mathbf{S}^{(1)}_{ij} \mathbf{r} \cdot \mathbf{r} + \mathbf{T}^{(1)}_{ij} \mathbf{r} \cdot \mathbf{r} + \mathbf{U}^{(1)}_{ij} \mathbf{r} \cdot \mathbf{r} = 0
\end{align*}
\]

\[\mathbf{S}_{ij} = \int_\Omega (\Delta \alpha_i)(\Delta \alpha_j) d\Omega \] (2a)

\[\mathbf{T}_{ij} = \int_\Omega (\alpha_i)(\alpha_j) d\Omega \] (2b)

The order of the symmetric matrices \( [S]_{11} \) and \( [S]_{22} \) correspond to the degrees of freedom of the nodal electric (\( \alpha_i \)) and magnetic (\( \beta_i \)) fields of the region (r) respectively. A similar definition applies to the \([T]\) matrices.

\([U] \) is a matrix of order \( (\alpha_ex\beta_m) \) and represents the physical coupling between the electric and magnetic fields at the dielectric boundaries of the region. The matrix entries are given by [3]

\[
U_{ij} = \int_\Omega \mathbf{r} \cdot \mathbf{r} \left( \frac{\partial \alpha_i}{\partial \tau} - \frac{\partial \beta_i}{\partial \tau} \right) d\tau
\] (2c)

\( \tau \) is an anticlockwise directed unit vector tangential to \( \xi \) and \( \kappa \) denotes the number...
of media interfaces along the boundary contour of a typical region. At cut-off and \([e_r] \) are \([h_r] \) decoupled.

**Cut-off space with dielectric filler between ridges.**

Modes in inhomogeneous waveguides may in general be described as quasi-LSE\(_{mn}\) or quasi-LSM\(_{mn}\) ones depending upon whether \(E_x\) or \(H_x\) is equal to zero as the dielectric loaded ridge waveguide is reduced to a rectangular one [11]. In general this structure supports hybrid modes, except in the case of the pure TE\(_{mn}\) modes which can exist when the dielectric extends across the full waveguide. The first three modes may be denoted as quasi-LSE\(_{10}\), LSE\(_{20}\), LSE\(_{11}\) or LSM\(_{01}\). One feature of this type of waveguide is that the separation between the cut-off frequencies of the dominant and first order quasi-LSE\(_{mn}\) modes is increased. While the calculations are restricted to the quasi-LSE\(_{10}\) and LSE\(_{20}\) modes, it is of note that it is possible for the quasi-LSE\(_{11}\) or LSM\(_{01}\) mode to correspond to the first higher order one [11].

At cut-off, \(\beta\) is zero, and the functional in (1) reduces to the sum of the two independent functionals associated with the planar problem region with top and bottom electric and magnetic walls respectively. Imposing appropriate symmetry planes and extremising the functional in conjunction with the FEM yields the quasi-LSE and LSM modes of the problem region. The discretisations in the dielectric and air regions are defined by \(p = 2\), \(m = 6\), \(n = 43\), \(n x m = 258\), and \(p = 2\), \(m = 6\), \(n = 49\), \(n x m = 294\), respectively. \(p\) is the degree of the interpolation polynomial within each finite element triangle, \(m\) is the number of nodes inside each finite element triangle, \(n\) is the number of triangles and \(n x m\) is the total number of nodes before assembly of the finite element mesh. The corresponding free (f) and prescribed (p) electric and magnetic nodes in the individual regions are \(f_e = 88\), \(f_h = 97\), \(p_e = 18\), \(p_h = 9\) and \(f_e = 91\), \(f_h = 120\), \(p_e = 29\), \(p_h = 0\). The dimension of the global matrices are \(210 \times 210\).

Figure 2 shows the regular variations in the normalised cut-off wavelength of the dominant quasi-LSE\(_{10}\) mode in a double ridge waveguide and also with a dielectric filler with a relative dielectric constant \((\varepsilon_r)\) equal to 9. Figure 3 indicates the separation between the quasi-LSE\(_{10}\) and LSE\(_{20}\) modes in each case. The discrete points in these illustrations correspond to the calculated values. The aspect ratio of the waveguide \((b/a)\) is 0.50 and the ridges are described by parametric values of \(d/b\) and \(s/a\) respectively.

**Figure 4: Propagation constant of dielectric loaded ridge waveguide, for \(s/a = 0.50\)**

**Figure 5: Comparison of theoretical and experimental propagation constants in square ridge waveguide with dielectric filler FEM, for \(\varepsilon_r = 3.47\), \(s/a = 0.25\), \(a = 8.55\)mm**

**Propagation constant with dielectric filler between ridges.**

The purpose of this section is to summarise some calculations on the propagation constant of the dominant quasi-LSE\(_{10}\) mode in double ridge waveguide with a dielectric filler between the top and bottom ridges. This is done for one waveguide topology and two different materials. The agreement between some calculations here and some obtained using the Transverse Resonance Method (TRM) in the case when the dielectric slab extends across the rectangular waveguide \((d/b = 1.0)\) has been verified as a preamble to
Double ridge waveguides proceeding with the main work. Figure 4 gives the main result obtained here. Figure 5 separately compares some calculations on a square ridge waveguide with a dielectric filler between its ridges and some measurement in the open literature [4]. This result suggests that the calculations represent a lower bound on experiment. The segmentation employed here is the same as that employed in the rectangular geometry. The experimental arrangement consisted of an aperture coupled half wave long cavity short-circuited at both ends.

Some calculations on a ridge waveguide with a dielectric filler extending over part of the open faces of the ridges have also been carried out for completeness sake. The topology considered here is illustrated in Figure 1b and some typical data is given in Figure 6.

Double ridge waveguide with dielectric tiles

Another ridge geometry of some interest is that in Figure 1c. It consists of a double ridge waveguide with dielectric tiles on the flat faces of the ridges. This geometry is defined by its normalised gap between the ridges, the normalised width of the ridges and the normalised thickness of a typical tile. A specification of the relative dielectric constant of the tiles completes its description [4].

A three component vector functional, which avoids the singularity at $\beta/k_0$ in the hybrid formulation, has been employed to investigate this topology [5-8]. The discretised functional in terms of the magnetic field is given by

$$ \mathbf{h}^\dagger \mathbf{D} \mathbf{h} = \int_{\Omega} \mathbf{E} \cdot \nabla \mathbf{h} \cdot \mathbf{d} \Omega $$

(4a)

$$ h_{t}^\dagger D_{t} h_{t} = \int_{\Omega} \mathbf{E} \cdot \nabla h_{t} \cdot h_{t} \cdot d \Omega $$

(4b)

$$ h_{z}^\dagger D_{z} h_{z} = \int_{\Omega} \left\{ E_{r} \left| \nabla_{s} h_{z} \right|^{2} - k_{0}^{2} \mathbf{h}_{z} \cdot \mathbf{d} \Omega \right\} $$

(4c)

$$ h_{z}^\dagger D_{z} h_{z} = \int_{\Omega} \left\{ \mathbf{E} \cdot \nabla h_{z} \cdot h_{z} \cdot d \Omega \right\} $$

(4d)

The matrices $[C]$ and $[D]$ are real symmetric and

$$ h_{t}^\dagger C_{t} h_{t} = \int_{\Omega} \left\{ E_{r} \left| \nabla_{s} h_{t} \right|^{2} - k_{0}^{2} \mathbf{h}_{t} \cdot \mathbf{d} \Omega \right\} $$

(4a)

$$ h_{z}^\dagger D_{z} h_{z} = \int_{\Omega} \left\{ E_{r} \left| \nabla_{s} h_{z} \right|^{2} - k_{0}^{2} \mathbf{h}_{z} \cdot \mathbf{d} \Omega \right\} $$

(4b)

The largest eigenvalue corresponds to the phase constant of the dominant mode at a specified wavenumber $k_0$. In order to avoid spurious modes the finite element discretisation has been based on a hybrid edge element technique in which edge and nodal elements are used for modelling the transverse and azimuthal components respectively [8].

Figure 7 shows some calculations on a square waveguide with dielectric tiles cemented on the open faces of the two ridges. The dimensions of the global matrices are 339 x 339 and the number of
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Ritz coefficients associated with the transverse and axial components are 242 and 97 respectively.

Voltage-current definition of impedance

The impedance of dielectric loaded double ridge waveguide is also of some interest. The purpose of this section is to give some calculations on the voltage-current definition of impedance of double ridge waveguide with a dielectric fillers between the ridges. The voltage is obtained in the usual way by evaluating the electric field along the magnetic symmetry plane extending between the two ridges.

\[
E_y = \frac{j \omega \eta_0}{(k_0 \varepsilon_r - \beta^2)} \left( \frac{\partial H_z}{\partial x} - \frac{j \beta}{k_0 \varepsilon_r - \beta^2} \frac{\partial E_z}{\partial y} \right)
\]

(6)

The current is separately obtained by integrating the magnetic field along the contour that is defined by the electric symmetry plane. This calculation may be undertaken by having recourse to \(H_x\) and \(H_y\) (equations 7a and 7b)

\[
H_x = \frac{-\beta}{k_0 \varepsilon_r - \beta^2} \left( \frac{\partial H_z}{\partial x} \right) + \frac{j \omega \eta_0}{(k_0 \varepsilon_r - \beta^2)} \left( \frac{\partial E_z}{\partial y} \right)
\]

(7a)

\[
H_y = \frac{-\beta}{k_0 \varepsilon_r - \beta^2} \left( \frac{\partial H_z}{\partial y} \right) - \frac{j \omega \eta_0}{(k_0 \varepsilon_r - \beta^2)} \left( \frac{\partial E_z}{\partial x} \right)
\]

(7b)

Unlike the homogenous problem region for which it is sufficient to calculate the impedance at infinite frequency and have recourse to the dispersion relationship to obtain that at the actual frequency, it is necessary in this instance to make the calculation at each individual frequency. Figure 8 indicates the result in the case of a dielectric constant of 9.0. Each curve is truncated at the cut-off frequency of the quasi-LSF20 mode in the respective topologies.

Conclusions

The Finite Element Method has been used in this paper to calculate the cut-off space and the propagation constant of a double ridge waveguide with various dielectric fillers between its ridges. This has been done for two geometries using single dielectric fillers between the ridges and one using dielectric tiles on the flat faces of the ridges. A comparison between theory and experiment in the case of a square arrangement suggests that the latter places a lower bound on the propagation constant of the waveguide. The impedance of one arrangement based on the voltage-current definition has also been calculated.

References